HOGESCHOOL ROTTERDAM

Training Digital Signal Processing

ELETDS02

FIR / IIR filters

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2 weeks ago

- **Signals** in real life are **continuous** and **analog**.
- Need to sample them to be able to process them digitally.
- They become **discrete-time digital signals**.
- Signals can be represented as sines/cosines with certain **frequencies**.
- Many problems are specified or solved in the **Fourier frequency domain**.
- We can switch between time and frequency domain with the **Fourier Transform**.

Last week

- Filters remove certain frequencies from a signal.
- Filters have **a transfer function** often specified in the **frequency domain**.
- We can implement the filter in the discrete-time domain by using the **IDTFT** (Inverse Discrete Time Fourier Transform).
- (Discrete-time) filters have several characteristic such as response shape (**LP, HP, BP, BS**), cut-off frequency and others.

FIR FILTERS AND WINDOWS

FIR Filter

- Last week we obtained a general formula for an FIR filter:

$$y[k] = \sum_{n=0}^{N} b_n \cdot x[k-n] = \vec{b} \cdot \vec{x}[k]$$

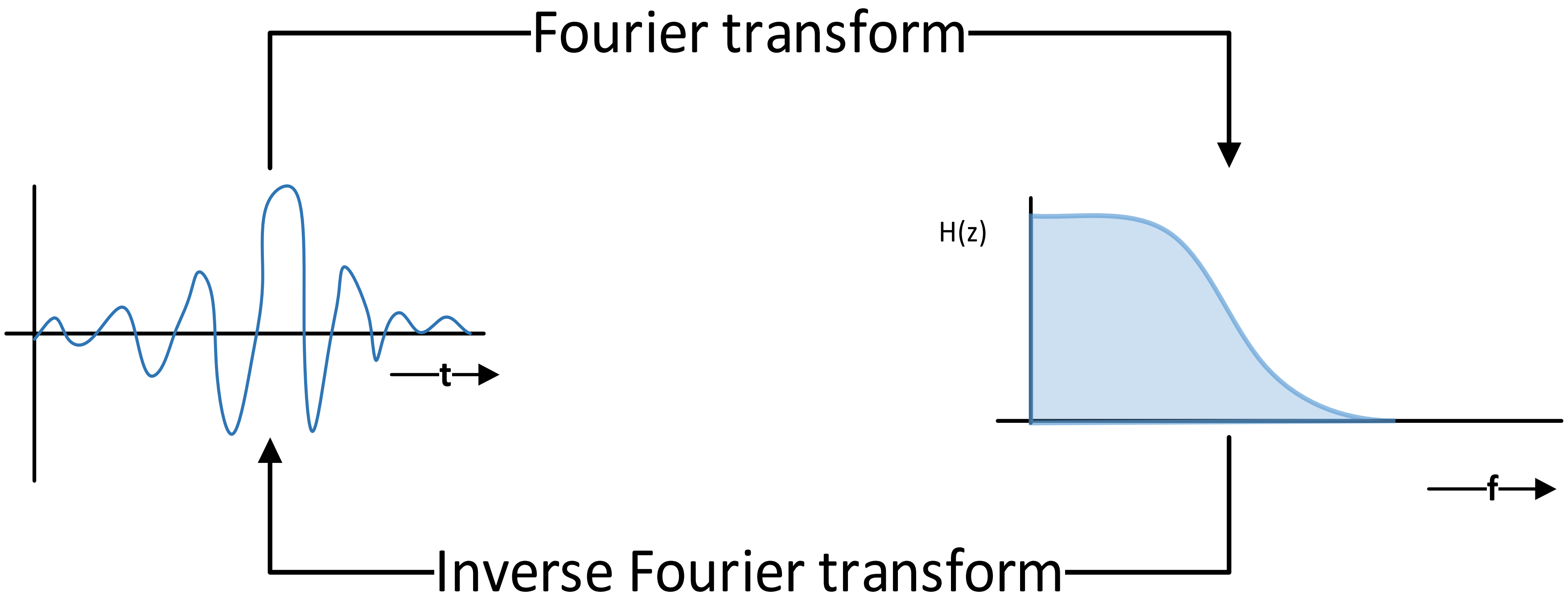
Vector with coefficients b_0 to b_N.

Vector with samples $x[k-N]$ to $x[k]$.

- How to get the coefficients?

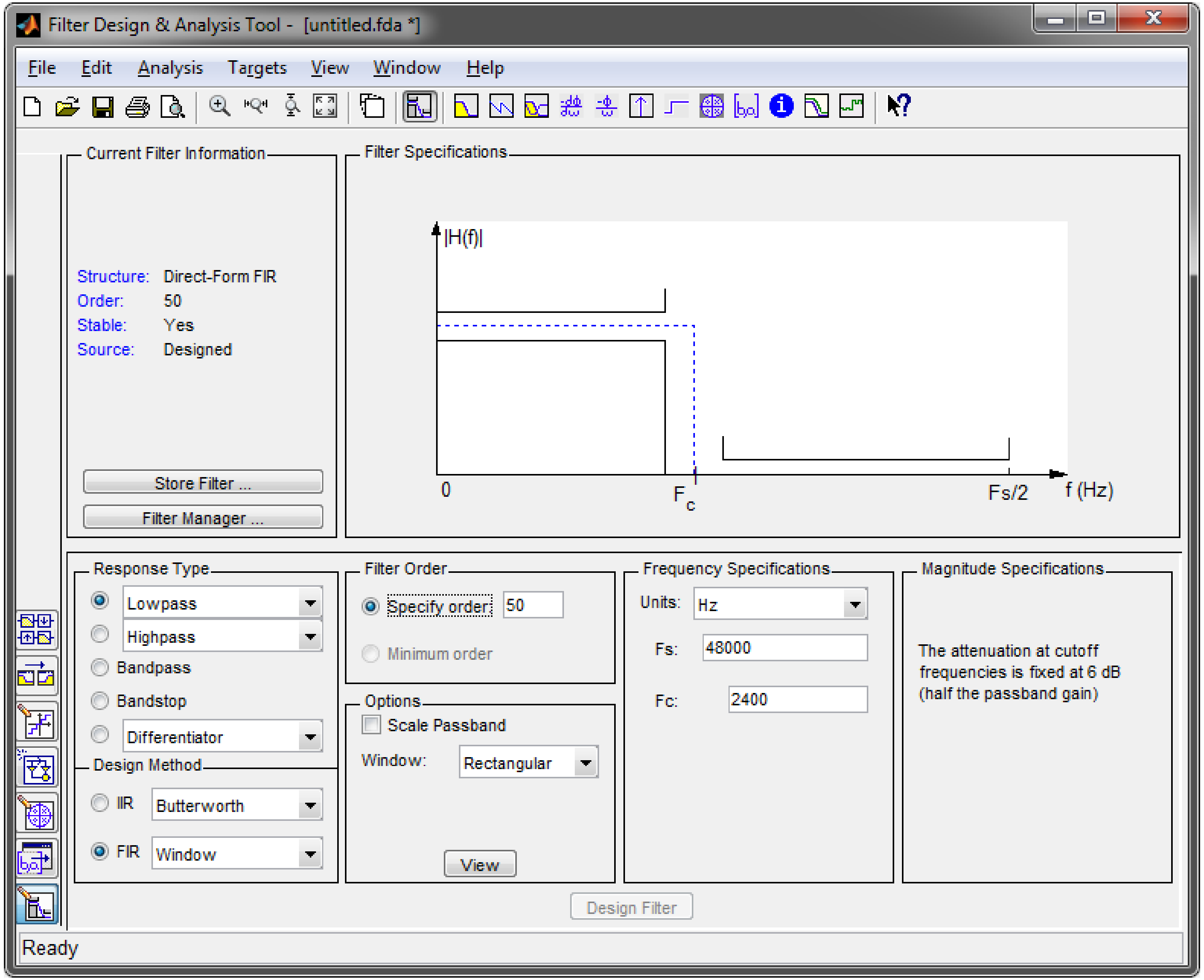
FIR filter coefficients

- Many methods:
  - **Window Design Method** (via IDTFT)
  - Frequency Sampling (also involves IDTFT)
  - Weighted Least Squares Method (need statistics :-( )
  - Some other methods



filterDesigner(1)

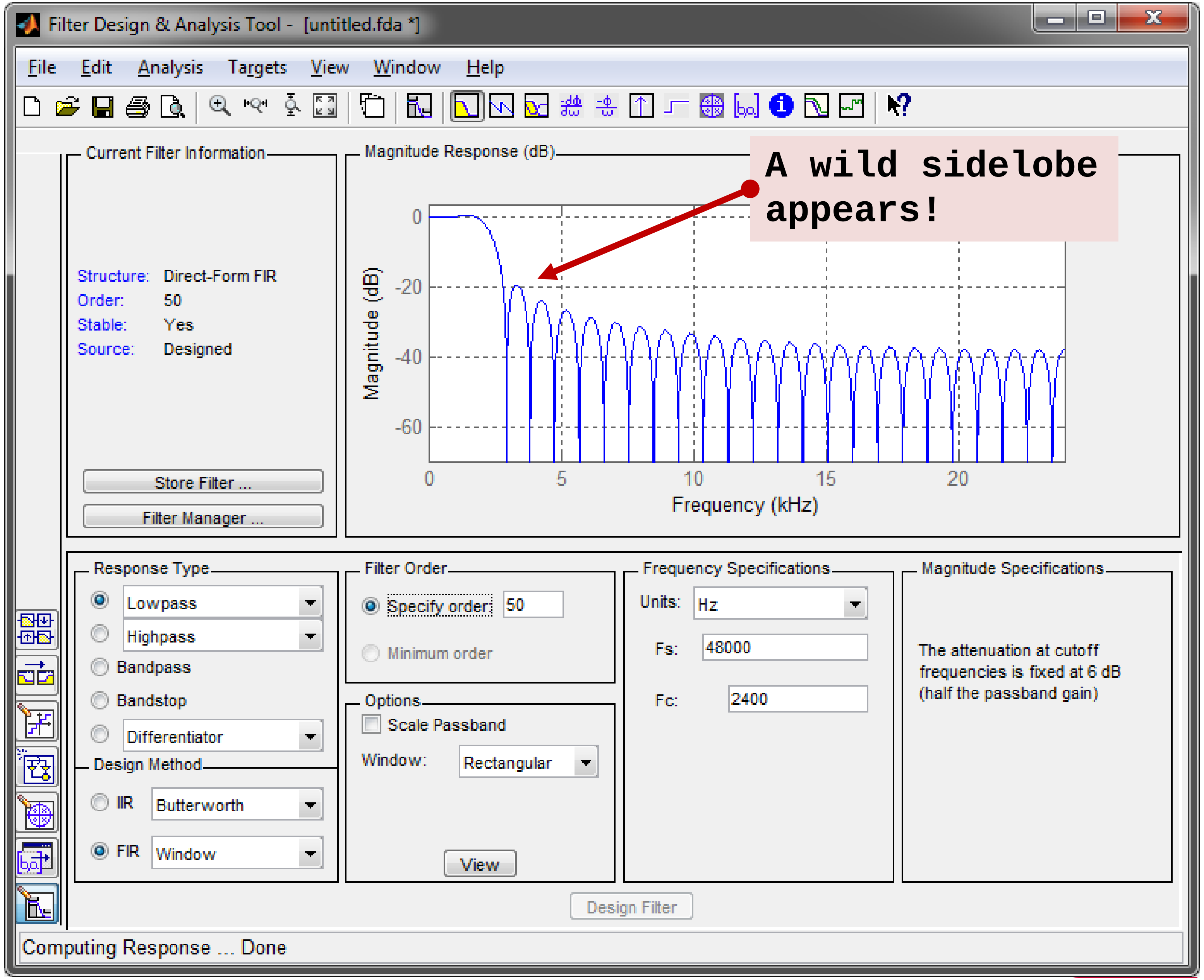
- MATLAB can apply the IDTFT for us.
- (It can also do many other methods)
- Use `filterDesigner`:
- Set relevant parameters.
- Click *“Design Filter”* …



filterDesigner (2)

- Resulting magnitude response is shown:

- But wait…



Windowing (1)

- The IDTFT is as follows:

$$x[n] = T\int_{\frac{-1}{2T_s}}^{\frac{1}{2T_s}} X(f)e^{j2\pi nfTs}df$$

- But... $n \in \mathbb{Z}$. (n can be any integer)
- We have an infinite number of coefficients that represents our filter in time. :-(
- To implement a filter in practice , we need to have a ***finite*** number of coefficients.
- The filter order specifies how many coefficients we use:

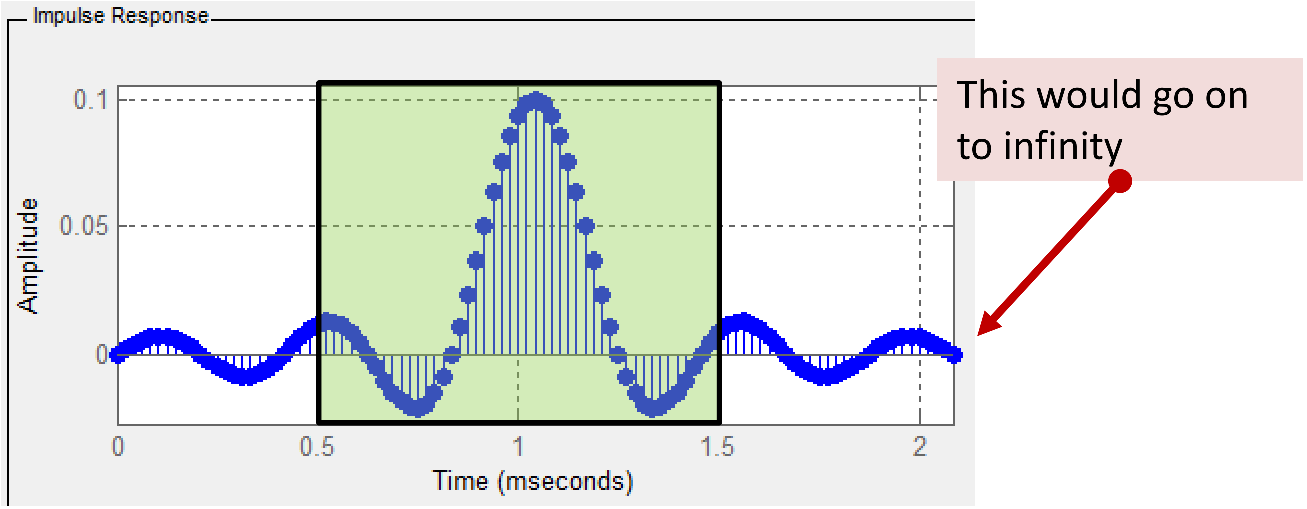
$$y[k] = \sum_{n=0}^{N} b_n \cdot x[k-n] = \vec{b} \cdot \vec{x}[k]$$

(For an N-th order FIR filter we need N samples.)

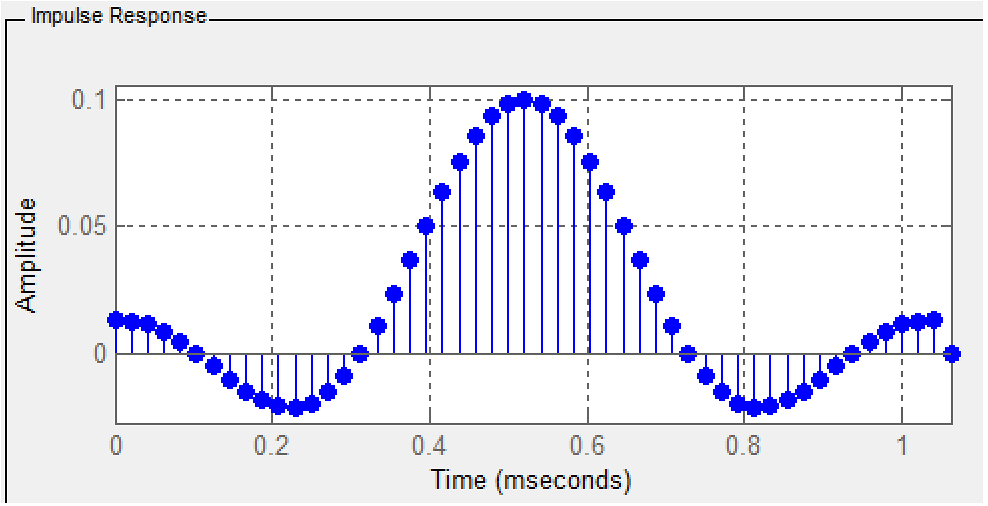
Windowing (2)

- Windowing is limiting the number of coefficients (to the desired filter order) in a certain way.

For example with a “rectangular” window:



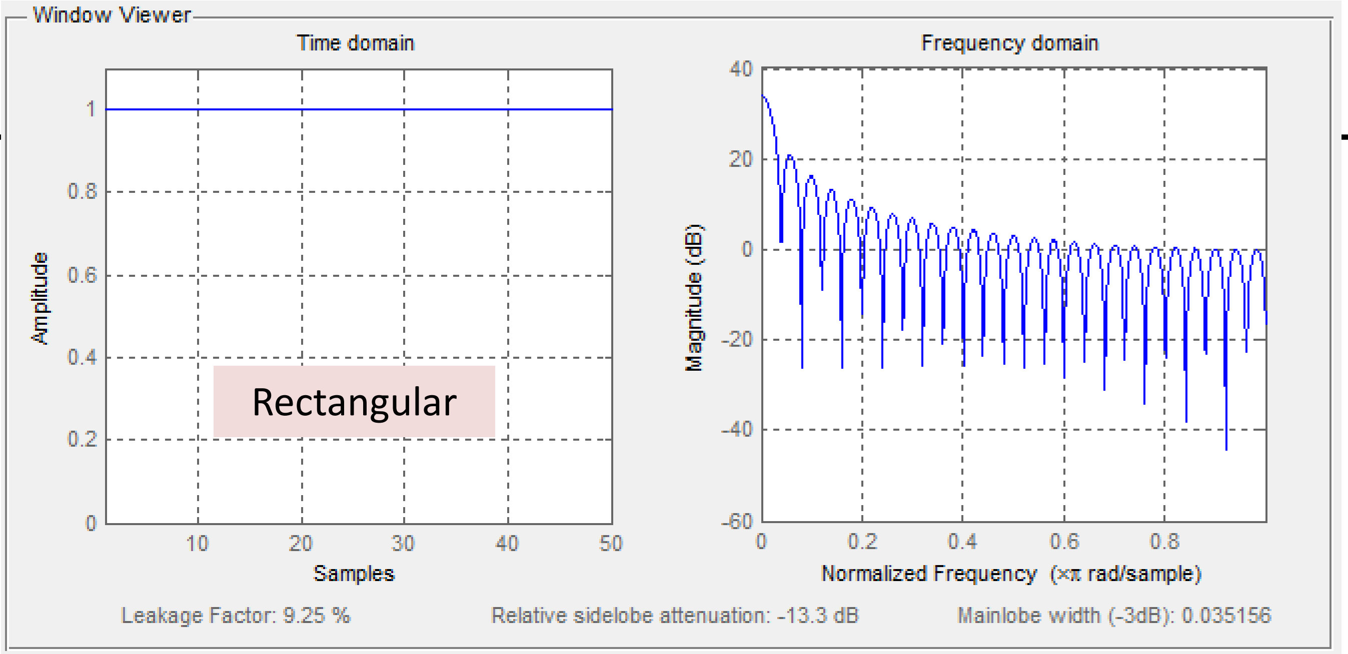
Windowed filter:



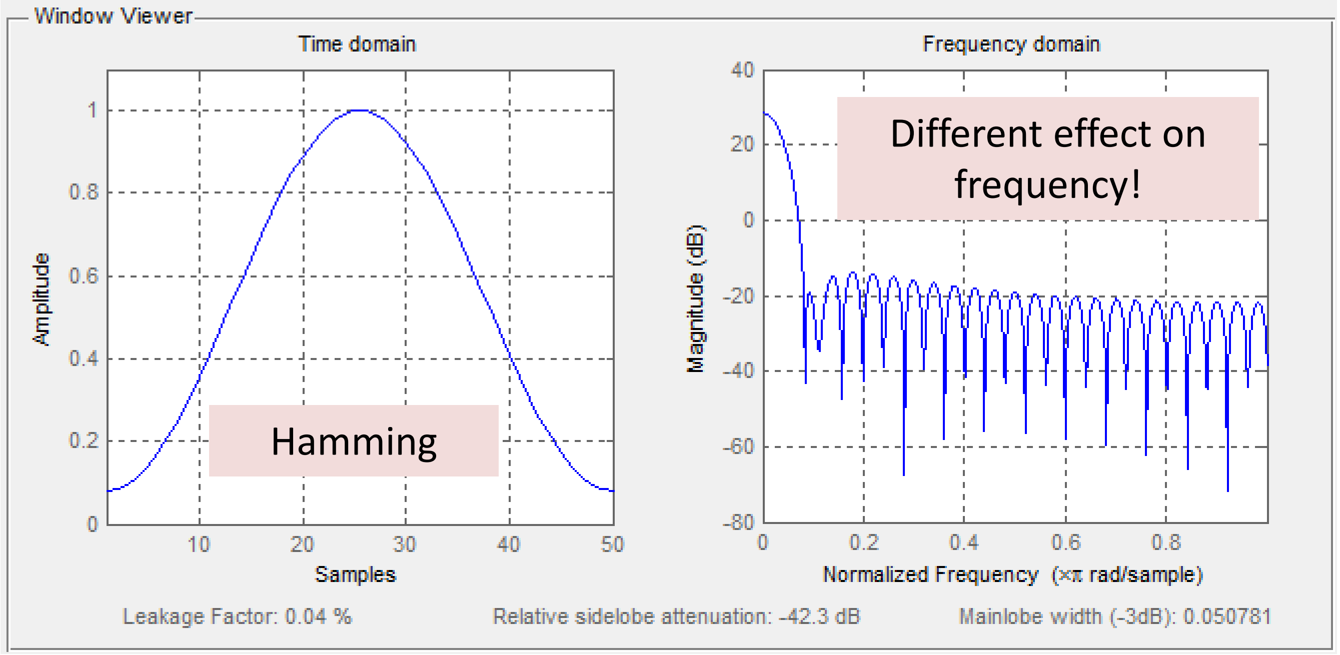
- We adjust the filter in time, what is the effect in frequency?

Windowing (3)

- A rectangular window is not the only window:

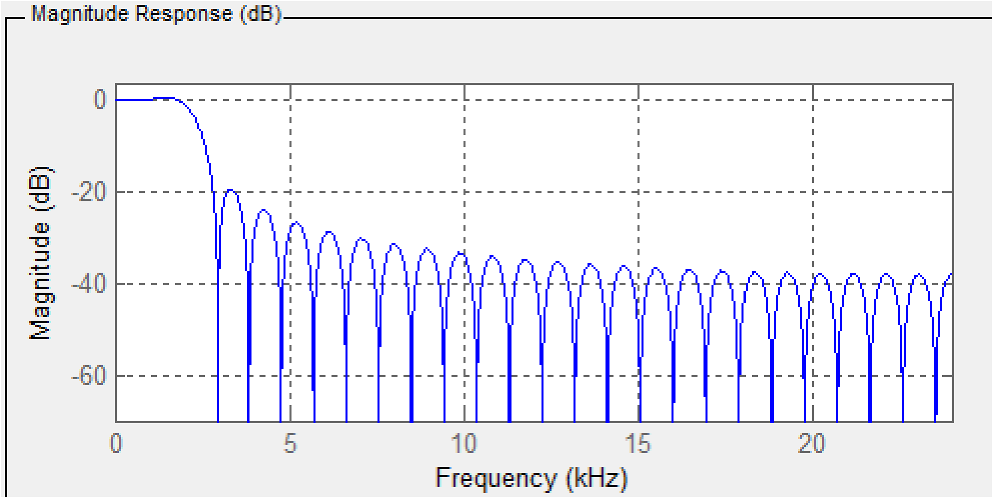


- We can let the coefficients at the edges go to zero more smoothly:

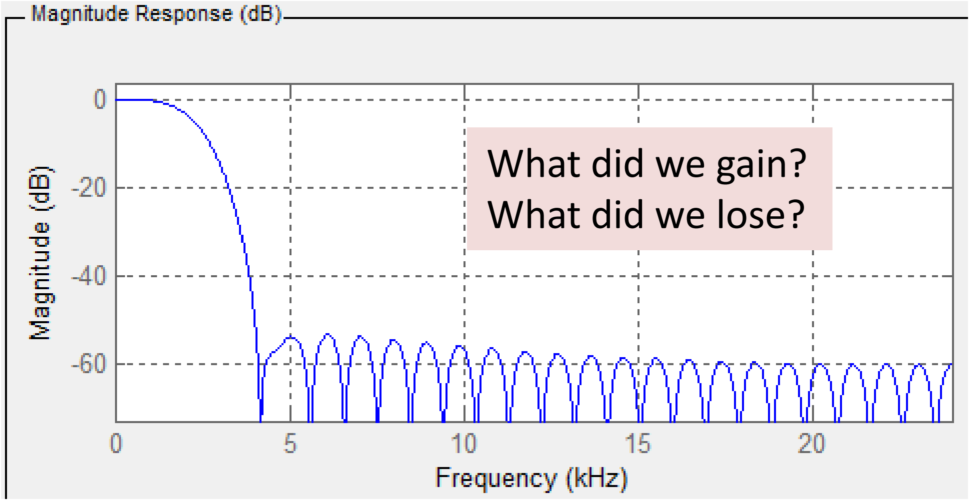


Windowing (4)

- LP filter with rectangular window:



- Same filter with Hamming window:



What did we gain?
What did we lose?

IIR FILTERS

IIR filters

- FIR is a non-recursive filter (no feedback).

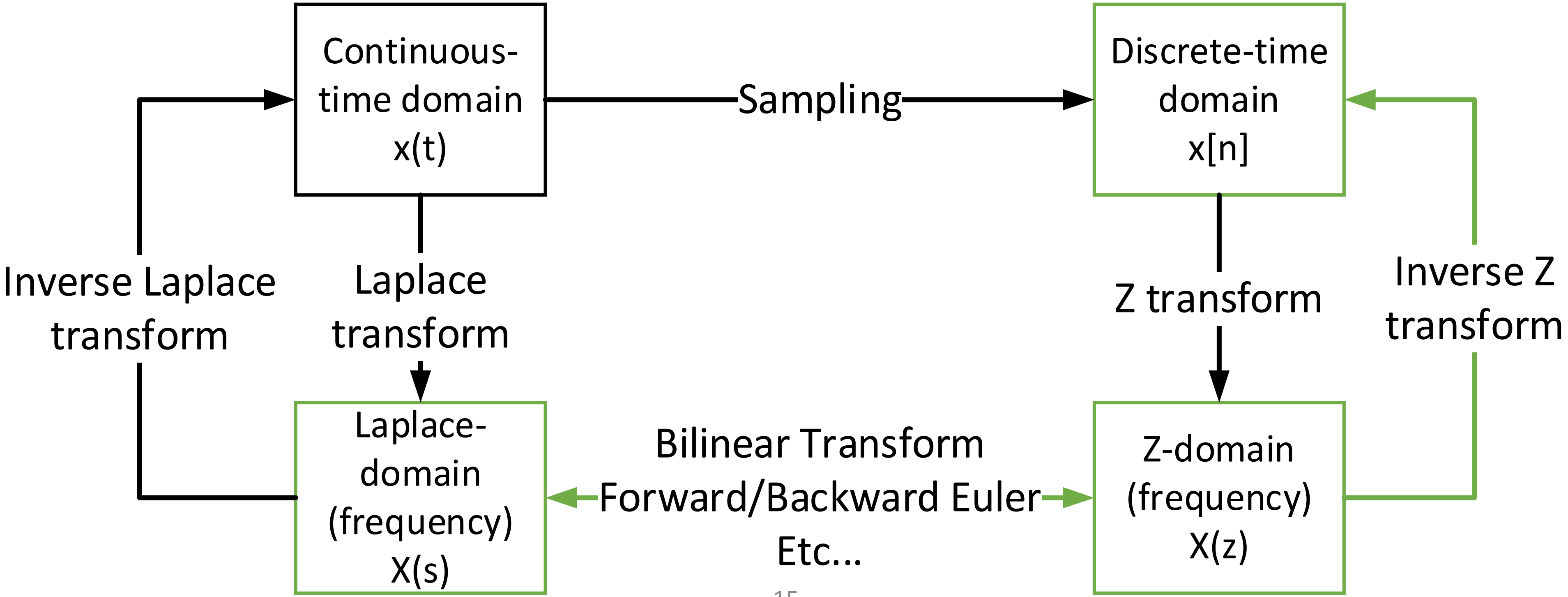
- Discrete-Time filters with feedback exist:

$$y[n] = \sum_{k=0}^{N} b_k \cdot x[n-k] - \sum_{i=1}^{M} a_i \cdot y[n-i]$$

- We call them Infinite Impulse Response filters (why?).
- The filter is some kind of difference equation.
- We have a special frequency domain for this called the Z-domain.
- It is very closely related to the Fourier frequency domain.

Even more transforms

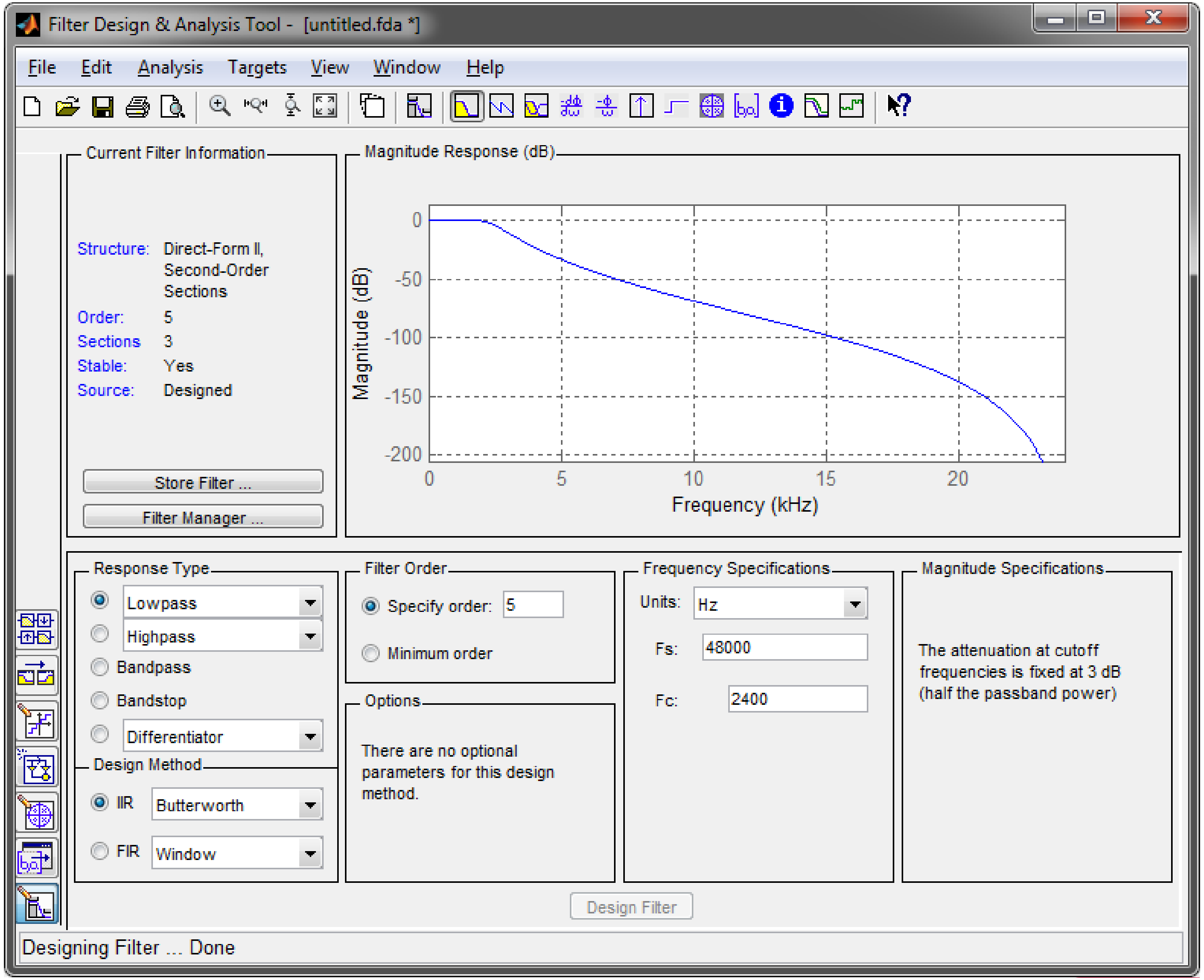
- IIR filters are much more effective with the same number of calculations (coefficients).
- However, because they contain feedback, the output can become **unstable**.
- They are often designed by looking at their well studied continuous-time equivalents.



IIR in filterDesigner

- `filterDesigner` does the math.

(A simple example is given in the lab handbook if you're interested.)



Comparison of FIR and IIR (1)

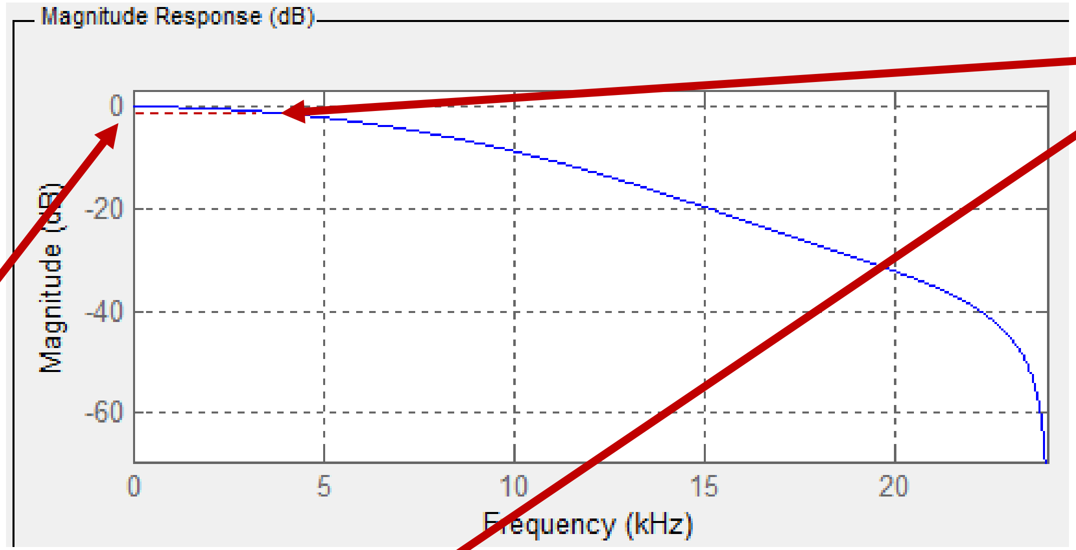
FIR:

5^{th} order

Fc = 2.4 kHz

Magnitude (dB)

≈ -3 dB



Fc

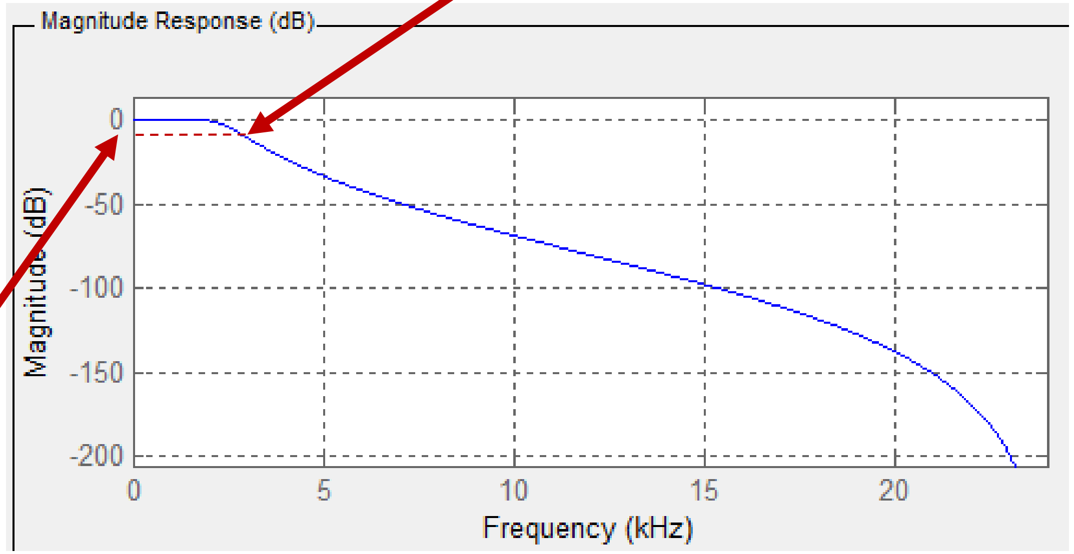
IIR (Butterworth):

5^{th} order

Fc = 2.4 kHz

Magnitude (dB)

≈ -15 dB



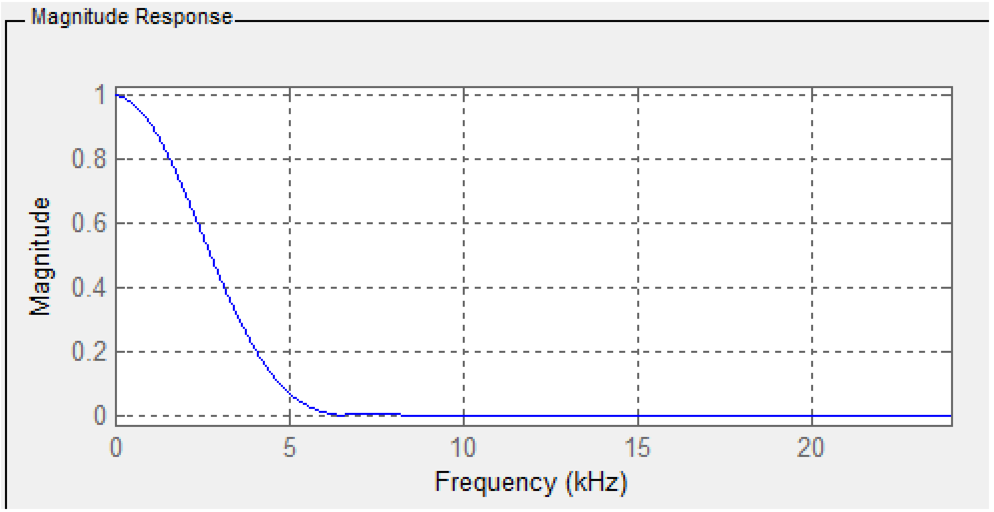
Comparison of FIR and IIR (2)

FIR (Hamming window):

Fc = 2.4 kHz

20th order

Magnitude

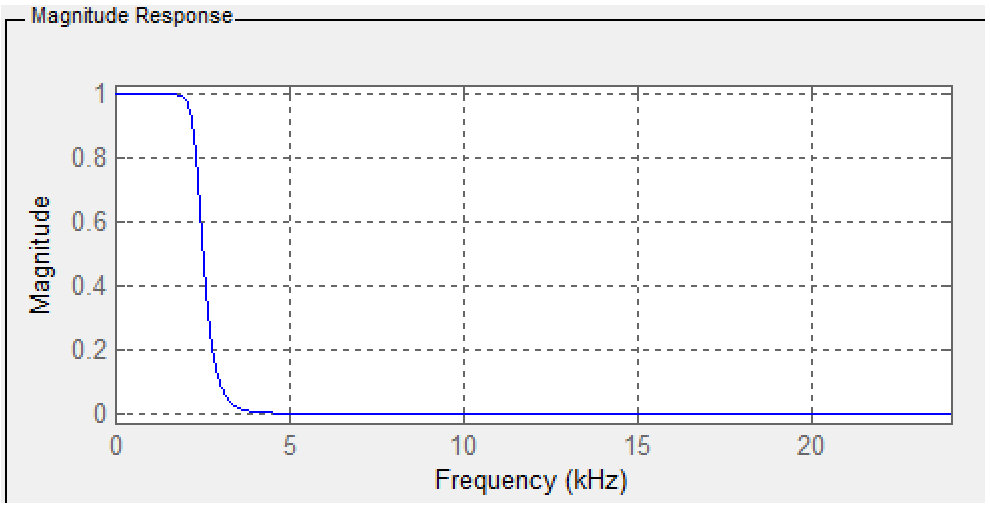


IIR (Butterworth):

Fc = 2.4 kHz

10th order

Magnitude

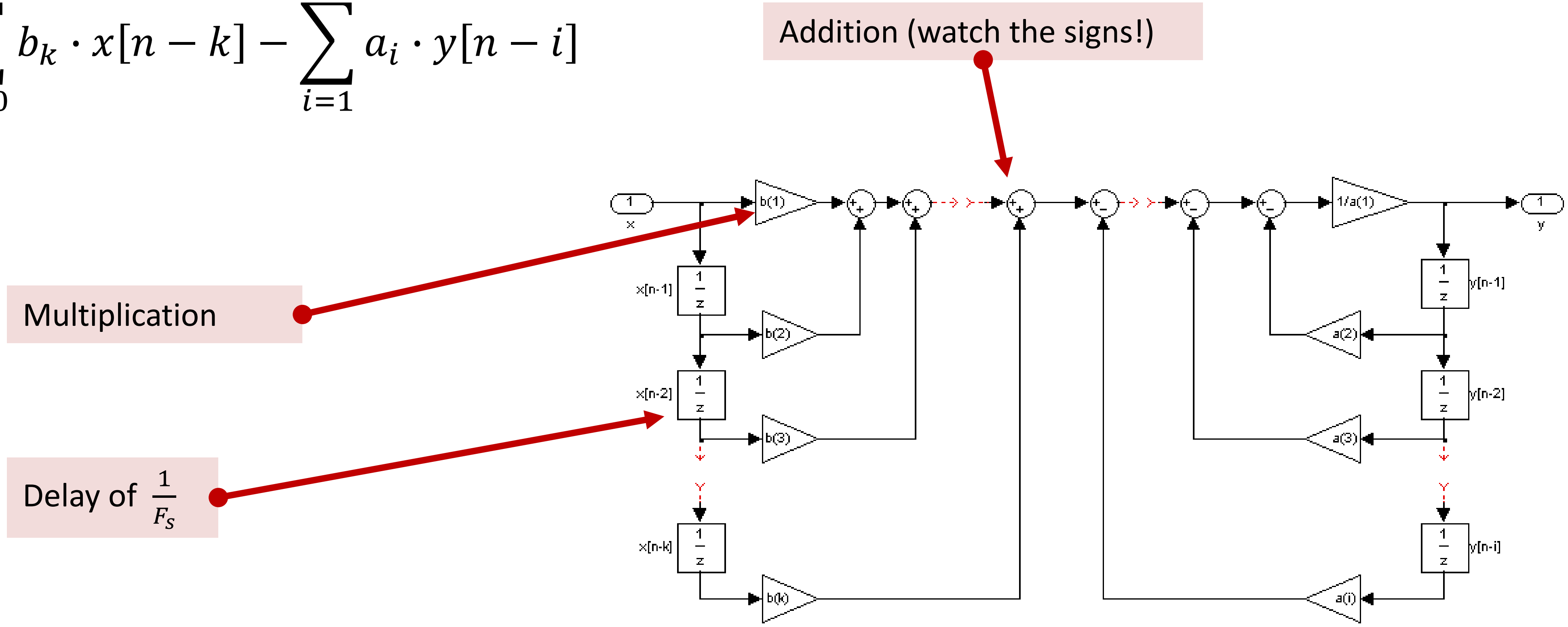


Roughly the same number of calculations

IIR filter structures

- For IIR filters different implementation structures exist (see lab handbook).
- Simplest form (Direct Form I):

$$y[n] = \sum_{k=0}^{N} b_k \cdot x[n-k] - \sum_{i=1}^{M} a_i \cdot y[n-i]$$



Summary

- The IDTFT gives us an infinite number of coefficients of our **FIR filter**.
- To implement the filter in practice we need to apply **windowing**.
- Rectangular windowing might introduce **unwanted effects** in the frequency domain.
- **Different window formulas** exist that try to keep certain unwanted effects to a minimum. (Experiment with these!)

- **IIR filters** contain feedback (or are recursive).
- With only a few coefficients good results can be achieved .
- Might be unstable.

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