Math 138 Physics Based Section
Assignment 6

(Q1) Determine if the following sequences converge and prove your result rigorously:
i)

$$a_n = \frac{1}{2^n}$$

ii)

$$a_n = \frac{1}{3^n}$$

iii)

$$a_n = \sin(n)\exp(-n)$$

iv) Give a general argument for why

$$a_n = \sin(n)$$

has no limit and explain why you would have trouble proving this result.

(Q2) Use limit rules to find the following limits.
i)

$$a_n = \exp(1/n)$$

ii)

$$\arctan(n^5)$$

iii) Show that

$$a_n = \sin(n\pi/5)$$

is bounded but does not have a limit. Explain why this is not a contradiction of one of the limit theorems.
iv)

$$a_n = 5 - \exp(-3n)$$

(Q3) Find counter examples for the following statements
i) If $a_n \longrightarrow L$ and $b_n \leq a_n$ then $b_n \longrightarrow L$.
ii) If $a_n \longrightarrow L$, $L > 0$ and $|b_n| \leq |a_n|$ then $b_n \longrightarrow L$.
iii) If b_n is bounded but has no limit, and $|a_n| \leq |b_n|$ then a_n has no limit.
iv) If a_n and b_n both have no limit then $c_n = a_n + b_n$ has no limit.
v) If a_n and b_n both have no limit then $c_n = a_n b_n$ has no limit.

(Q4) Consider a unit circle. Begin at the point $(1, 0)$ and move a fixed distance $l > 0$ along the circle. Record the new point. Now repeat the process. For what values of l will you eventually hit every point on the circle?

(Q5)i) Consider the sequence of functions

$$f_n(x) = x^{2n}$$

For any fixed input (say $x = 1$ or $x = -5$) the above is a sequence of numbers. Find the limit for various values of x (or in other words for various inputs) just as we did for x^n in class.
ii) Repeat for

$$f_n(x) = (2x)^n$$

(Q6) i) Consider the proof that $a_n \longrightarrow L$. If for $\epsilon_1 > 0$ we can find N_1 so that $n > N_1$ ensures that $|a_n - L| < \epsilon_1$ and for $\epsilon_2 = \epsilon_1/2$ we can find N_2 so that $n > N_2$ ensures that $|a_n - L| < \epsilon_2$ show that $N_2 \geq N_1$.
Prove that if $a_n \longrightarrow 1$ and $b_n \longrightarrow 2$ then $c_n = a_n + b_n \longrightarrow 3$.
ii) Choose a target $\epsilon = m > 0$ and use the definition of limit for a_n with $\epsilon = m/2$
iii) Repeat for b_n.
iv) Now use parts ii) and iii) along with the triangle inequality to find N so that $|a_n + b_n - 3| < \frac{m}{2} + \frac{m}{2}$ for $n > N$.

(Q7) Find the following series expansions and indicate for what input points the series converges i)

$$\frac{1}{3-x}$$

ii)

$$\frac{1}{2-3x}$$

iii)

$$\frac{1}{3+x}$$

(Q8) DO NOT HAND IN (could be very useful for test 2)
i) page 83 exercise 2
ii) page 83 exercise 4
iii) page 83 exercise 5