

Submarines

Two submarines are approaching each other on a collision course, the first with a speed v_1 and the second with a speed v_2 . The speed of sound in sea water is v_{sea} .

a) The first submarine emits a sonar pulse of frequency f_1 . What is the frequency, f_2 , of this pulse as measured by the second submarine?

$$f_2 = \left(\frac{v_{\text{sea}} + v_2}{v_{\text{sea}} - v_1} \right) \cdot f_1$$

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b) The sonar pulse is reflected by the second submarine and detected by the first submarine at a frequency f_3 . In terms of the first submarine's speed, v_1 , the speed of sound in seawater, v_{sea} , and the original frequency, f_1 , find the speed of the second submarine, v_2 .

$$f_3 = \left(\frac{v_{\text{sea}} + v_1}{v_{\text{sea}} - v_2} \right) \cdot \left(\frac{v_{\text{sea}} + v_2}{v_{\text{sea}} - v_1} \right) \cdot f_1 \text{ or } f_2 = \frac{v_{\text{sea}}}{v_{\text{sea}} - (v_1 + v_2)} f_1, f_3 = \frac{v_{\text{sea}}}{v_{\text{sea}} - (v_1 + v_2)} f_2$$

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solving for v_2 , we get: $v_2 = v_{\text{sea}} \left(1 - \sqrt{\frac{f_1}{f_3}} \right) - v_1$

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c) What is the $v_1 \rightarrow 0$ limit to your answer to part (b)?

$$\lim_{v_1 \rightarrow 0} v_2 = v_{\text{sea}} \left(1 - \sqrt{\frac{f_1}{f_3}} \right)$$

d) What is the $f_3 \rightarrow f_1$ limit to your answer to part (b)?

$$\lim_{f_1'' \rightarrow f_1} v_2 = -v_1$$