MATH 136 **Assignment 8** **Fall/05**

Due: Thursday Nov. 24/05
(Show details of your work. Grade is out of 28.)

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1 page 243 #14

4 marks

(a) Since A,B are row equivalent,

$$\forall x \in \Re^5, Ax = b \Longleftrightarrow Bx = b.$$

Therefore

$$x \in \mathrm{Nul}\,(\mathrm{A}) \Longleftrightarrow \mathrm{Ax} = 0 \Longleftrightarrow \mathrm{Bx} = 0.$$

The set of columns of B has three linearly independent columns, i.e. there are two linearly independent columns in Nul (B). We can find these by observation. Suppose $x \in \Re^5, x \in \mathrm{Nul}\,(\mathrm{B})$. By row 4, we find $x_5 = 0$ necessarily. Continuing such an argument we find that

$$\mathrm{Nul}\,(\mathrm{A}) = \mathrm{Nul}\,(\mathrm{B}) = \left\{ \mathrm{t} \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mathrm{s} \begin{pmatrix} -4 \\ 0 \\ \frac{7}{5} \\ 1 \\ 0 \end{pmatrix} : \mathrm{s}, \mathrm{t} \in \Re \right\}$$

Here $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ \frac{7}{5} \\ 1 \\ 0 \end{pmatrix} \right\}$ s a basis for Nul (B) = Nul (A).

(b) Row reduction does *not* change linear independence or dependence of columns in a matrix. Therefore, from observation on B, columns 1,3,5 or columns 2,4,5 etc... can be chosen as a basis for the column space of A. Note that the column space itself *does* change. So we have to use the columns of A!

$$\mathrm{Col}\,(\mathrm{A}) = \mathrm{span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ -5 \\ -5 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 5 \\ -2 \end{pmatrix} \right\}.$$

The set of these three vectors forms the required basis.

2 page 243 #16

4 marks

We just need to do the row reduction for the matrix $[v_1, v_2, v_3, v_4, v_5]$, and its pivot columns are the basis for $\mathrm{span}\{v_1, v_2, v_3, v_4, v_5\}$. So we use matlab to do row reduction:

```
A=[1 -2 6 5 0;0 1 -1 -3 3;0 -1 2 3 -1;1 1 -1 -4 1];
A(2,:)=A(2,:)-A(1,:)*A(2,1)/A(1,1)
A(3,:)=A(3,:)-A(3,1)*A(1,:)/A(1,1)
A(4,:)=A(4,:)-A(4,1)*A(1,:)/A(1,1)
A(3,:)=A(3,:)-A(3,2)*A(2,:)/A(2,2)
A(4,:)=A(4,:)-A(4,2)*A(2,:)/A(2,2)
A(4,:)=A(4,:)-A(4,3)*A(3,:)/A(3,3)
```

The result is as following:

```
A =

     1    -2     6     5     0
     0     1    -1    -3     3
     0    -1     2     3    -1
     1     1    -1    -4     1


A =

     1    -2     6     5     0
     0     1    -1    -3     3
     0    -1     2     3    -1
     1     1    -1    -4     1


A =
```

```
     1    -2     6     5     0
     0     1    -1    -3     3
     0    -1     2     3    -1
     0     3    -7    -9     1


A =

     1    -2     6     5     0
     0     1    -1    -3     3
     0     0     1     0     2
     0     3    -7    -9     1


A =

     1    -2     6     5     0
     0     1    -1    -3     3
     0     0     1     0     2
     0     0    -4     0    -8


A =

     1    -2     6     5     0
     0     1    -1    -3     3
     0     0     1     0     2
     0     0     0     0     0
```

Thus v_1, v_2, v_5 are a basis of span$\{v_1, v_2, v_3, v_4, v_5\}$. (Notice there are many different combinations for a basis of span$\{v_1, v_2, v_3, v_4, v_5\}$. We just need to ensure there are pivots on each column vector.)

3 page 245 #32

4 marks

Suppose that the set $\{T(v_1), T(v_2), \ldots, T(v_p)\}$ is linearly dependent, i.e. suppose that $\exists c_1, c_2, \ldots c_p \in \Re$, which are NOT all zero, and the (nontrivial) linear combination $c_1T(v_1) + c_2T(v_2) \ldots + c_pT(v_p) = 0$. Since T is a linear transformation, we see that

$$T(c_1v_1 + c_2v_2 \ldots + c_pv_p) = c_1T(v_1) + c_2T(v_2) \ldots + c_pT(v_p) = 0$$

But $T(0) = 0$. Therefore, since T is one-to-one, we get

$$c_1v_1 + c_2v_2 \ldots + c_pv_p = 0.$$

Since $c_1, c_2, \dots c_p$ are not all zero, we conclude that $\{v_1, v_2, \dots v_p\}$ is a linearly dependent set.

4 page 245 #34

4 marks

By inspection,

$$p_1(t) + p_2(t) - 2p_3(t) = 1 + t^2 + 1 - t^2 - 2 = 0.$$

And we have p_1, and p_2 are linearly independent. Thus they (as a set) form a basis. (Note: the sets $\{p_1, p_3\}$, $\{p_2, p_3\}$ can also be used to form a basis.)

5 page 254 #6

3 marks

We need to solve for y in:

$$\begin{pmatrix} b_1 & b_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = x.$$

Then x is the linear combination of b_1, b_2 with coordinates y_1, y_2.

$$y = \begin{pmatrix} 1 & 5 \\ -2 & -6 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -6 & -5 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -24 \\ 8 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

6 page 254 #10

3 marks

Suppose that M is the change-of-coordinate matrix from basis β to standard basis as given by the columns of the identity matrix $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, i.e. $[x]_E = M[x]_\beta$, i.e. $[x]_E$ is a linear combination of the columns of M. Therefore, M is the matrix formed from the vectors of the basis β, i.e.

$$M = \begin{pmatrix} 3 & 2 & 8 \\ -1 & 0 & -2 \\ 4 & -5 & 7 \end{pmatrix}$$

7 page 254 #16

6 marks

(a) True.

If β is the standard basis for $\Re^n$, $P_\beta = I$, the identity matrix in $\Re^n$, i.e.

$$[x]_\beta = P_\beta x = Ix = x$$

(b) False.

The mapping: $x \Longrightarrow [x]_\beta]$ is called the *coordinate mapping*.

(c) True

$P := \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ is a plane of $\Re^3$, which is isomorphic to $\Re^2$, since we can define the one-to-one, onto linear transformation T from P onto $\Re^2$:

$$T\begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

.

(In fact, this is true for any plane in $\Re^3$ since every plane is given by $P = \operatorname{span}\{v_1, v_2\}$ for two linearly independent vectors in $\Re^3$. We can then use these two vectors to form a basis β for the plane P and the linear transformation for the isometry is the coordinate mapping on the subspace $V = P$.