

Wave on a vertical string

A uniform rope of mass m and length ℓ hangs vertically from the ceiling.

a) Find the speed of a wave on the string as a function of x , the distance from the lower end of the string.

$$V_w(x) = \sqrt{\frac{T(x)}{\rho}}$$
$$T(x) \propto x$$
$$T(x) = kx$$
$$T(x) = \frac{mg}{\ell}x$$
$$V_w(x) = \sqrt{gx}$$

$$v_w(x) = \underline{\hspace{2cm}}$$

b) Find the time taken for a wave to travel from the bottom to the top of the string. (Hint: This will involve an integral.)

Find time it takes to travel a distance of ℓ , at $v_w(x) = \sqrt{gx}$

$$V = \frac{dx}{dt}$$
$$dt = \frac{dx}{V}$$
$$\Rightarrow t = \int_0^\ell \frac{1}{V} dx$$

$$t_{\ell,m} = 2\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow t = 2\sqrt{\frac{\ell}{g}}$$

c) Find the time taken for a wave to travel from the bottom to the top of the string if the mass of the string is doubled.

t does not depend on mass.

$$t_{\ell,2m} = 2\sqrt{\frac{\ell}{g}}$$

d) Find the time taken for a wave to travel from the bottom to the top of the string if the length of the string is doubled.

$$t_{2\ell,m} = 2\sqrt{\frac{2\ell}{g}}$$