MATH 136 **Assignment 4** **Fall/05**

Due: Thursday Oct. 13/05
(Show details of your work. Grade is out of 37.)

Contents

1 page 56 #38

4 marks

No. First, if $Ax = y$ has no solution, then the RREF has a row: $\begin{pmatrix} 0 & 0 & 0 & b \end{pmatrix}, b \neq 0$, i.e. the 4-th column is a pivot column and the RREF of A has at most two pivot columns. Then the homogeneous linear system Ax=0 will have nontrivial a solution v_h. So if Ax=z has a solution p, then $p + v_h$ which doesn't equal p will be another solution. So Ax=z never has unique solution.

2 page 71 #6

3 marks

row reduction for the matrix; matlab program is:

```
A=[
-4 -3 0
0 -1 4
1 0 3
5 4 6]
A(1,:)=A(1,:)/A(1,1);
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(4,:)=A(4,:)-A(4,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(4,:)=A(4,:)-A(4,2)*A(2,:)
```

the result is:

```
A =

    -4    -3     0
     0    -1     4
     1     0     3
     5     4     6


A =

    1.0000    0.7500         0
         0   -1.0000    4.0000
         0   -0.7500    3.0000
    5.0000    4.0000    6.0000


A =

    1.0000    0.7500         0
         0   -1.0000    4.0000
         0   -0.7500    3.0000
         0    0.2500    6.0000


A =

    1.0000    0.7500         0
         0    1.0000   -4.0000
         0   -0.7500    3.0000
         0    0.2500    6.0000


A =
```

```
    1.0000    0.7500         0
         0    1.0000   -4.0000
         0         0         0
         0    0.2500    6.0000


A =

    1.0000    0.7500         0
         0    1.0000   -4.0000
         0         0         0
         0         0    7.0000
```

so we see each colum has a pivot, so the three vectors are linearly indepedent.

3 page 71 #10

3 marks

The set $\{v_1, v_2, v_3\}$ is linearly indepedent if and only if the matrix $A = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$ has a pivot in each column. so we do row reduction to A.

```
h=sym('h','real')
A=[
1 -2 2
-5 10 -9
-3  6 h]
A(1,:)=A(1,:)/A(1,1);
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
```

the result is:

```
h =

h


A =

[  1, -2,  2]
[ -5, 10, -9]
```

```
[ -3,  6,  h]


A =

[   1,  -2,   2]
[  -5,  10,  -9]
[   0,   0, h+6]


A =

[     1,    -2,     2]
[  -1/2,     1, -9/10]
[     0,     0,   h+6]


A =

[     1,    -2,     2]
[  -1/2,     1, -9/10]
[     0,     0,   h+6]
```

Note that the first two columns are linearly dependent (column 2 is a multiple of column 1) and column 3 is never a multiple of column 2. Therefore, The v_3 is never in the span of the first two vectors (part a). But, the set is linearly dependent for all values of h.

4 page 71 #22

8 marks

(a) **True**. First, if one of the vectors is 0, then the result is obvious. And if both are not zero: i.e. $\alpha_1 v_1 + \alpha_2 v_2 = 0$ and the scalar $\alpha_1 \neq 0$ if and only if $v_1 = -\frac{\alpha_2}{\alpha_1} v_2$.

(b) **false** For example, suppose one of them is the zero vector.

(c) **true**, by the definition of span $z = \alpha x + \beta y$ means $z - \alpha x - \beta y = 0$.

(d) **False**, e.g. set containing 0 in $\Re$.

5 page 72 #32

3 marks

we have:

$$1 \begin{pmatrix} 4 \\ -7 \\ 9 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix} = 0$$

so $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ will be a solution to Ax=0

6 page 72 #42

4 marks

We need to find the largest linearly indepedent set of columns of A. So we perform row reduction for the matrix:

```
A=[
12 10 -6 -3 7 10
-7 -6 4 7 -9 5
9 9 -9 -5 5 -1
-4 -3 1 6 -8 9
8 7 -5 -9 11 -8]
A(1,:)=A(1,:)/A(1,1);
A(2,:)=A(2,:)-A(2,1)*A(1,:)
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(4,:)=A(4,:)-A(4,1)*A(1,:)
A(5,:)=A(5,:)-A(5,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(4,:)=A(4,:)-A(4,2)*A(2,:)
A(5,:)=A(5,:)-A(5,2)*A(2,:)
A(3,:)=A(3,:)/A(3,4)
A(4,:)=A(4,:)-A(4,4)*A(3,:)
A(5,:)=A(5,:)-A(5,4)*A(3,:)
```

the result is as:

```
A =

    12    10    -6    -3     7    10
    -7    -6     4     7    -9     5
     9     9    -9    -5     5    -1
    -4    -3     1     6    -8     9
     8     7    -5    -9    11    -8


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0   -0.1667    0.5000    5.2500   -4.9167   10.8333
    9.0000    9.0000   -9.0000   -5.0000    5.0000   -1.0000
   -4.0000   -3.0000    1.0000    6.0000   -8.0000    9.0000
    8.0000    7.0000   -5.0000   -9.0000   11.0000   -8.0000
```

```
A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0   -0.1667    0.5000    5.2500   -4.9167   10.8333
         0    1.5000   -4.5000   -2.7500   -0.2500   -8.5000
   -4.0000   -3.0000    1.0000    6.0000   -8.0000    9.0000
    8.0000    7.0000   -5.0000   -9.0000   11.0000   -8.0000


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0   -0.1667    0.5000    5.2500   -4.9167   10.8333
         0    1.5000   -4.5000   -2.7500   -0.2500   -8.5000
         0    0.3333   -1.0000    5.0000   -5.6667   12.3333
    8.0000    7.0000   -5.0000   -9.0000   11.0000   -8.0000


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0   -0.1667    0.5000    5.2500   -4.9167   10.8333
         0    1.5000   -4.5000   -2.7500   -0.2500   -8.5000
         0    0.3333   -1.0000    5.0000   -5.6667   12.3333
         0    0.3333   -1.0000   -7.0000    6.3333  -14.6667


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0    1.0000   -3.0000  -31.5000   29.5000  -65.0000
         0    1.5000   -4.5000   -2.7500   -0.2500   -8.5000
         0    0.3333   -1.0000    5.0000   -5.6667   12.3333
         0    0.3333   -1.0000   -7.0000    6.3333  -14.6667


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0    1.0000   -3.0000  -31.5000   29.5000  -65.0000
         0         0    0.0000   44.5000  -44.5000   89.0000
         0    0.3333   -1.0000    5.0000   -5.6667   12.3333
         0    0.3333   -1.0000   -7.0000    6.3333  -14.6667
```

```
A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0    1.0000   -3.0000  -31.5000   29.5000  -65.0000
         0         0    0.0000   44.5000  -44.5000   89.0000
         0         0    0.0000   15.5000  -15.5000   34.0000
         0    0.3333   -1.0000   -7.0000    6.3333  -14.6667


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0    1.0000   -3.0000  -31.5000   29.5000  -65.0000
         0         0    0.0000   44.5000  -44.5000   89.0000
         0         0    0.0000   15.5000  -15.5000   34.0000
         0         0    0.0000    3.5000   -3.5000    7.0000


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0    1.0000   -3.0000  -31.5000   29.5000  -65.0000
         0         0    0.0000    1.0000   -1.0000    2.0000
         0         0    0.0000   15.5000  -15.5000   34.0000
         0         0    0.0000    3.5000   -3.5000    7.0000


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0    1.0000   -3.0000  -31.5000   29.5000  -65.0000
         0         0    0.0000    1.0000   -1.0000    2.0000
         0         0   -0.0000         0         0    3.0000
         0         0    0.0000    3.5000   -3.5000    7.0000


A =

    1.0000    0.8333   -0.5000   -0.2500    0.5833    0.8333
         0    1.0000   -3.0000  -31.5000   29.5000  -65.0000
         0         0    0.0000    1.0000   -1.0000    2.0000
         0         0   -0.0000         0         0    3.0000
         0         0    0.0000         0    0.0000    0.0000
```

So columns $1, 2, 4, 6$ are pivot columns, and they form a largest linearly indepedent set of columns of A. Notice we could also replace column 2 with column 3; or we could replace column 4 with column 5. Suppose we choose columns 1,2,,4,6.

```
A=[
12 10  -3 10 0
-7 -6  7  5 0
9 9  -5 -1 0
-4 -3  6  9 0
8 7  -9  -8 0]
A(1,:)=A(1,:)/A(1,1);
A(2,:)=A(2,:)-A(2,1)*A(1,:)
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(4,:)=A(4,:)-A(4,1)*A(1,:)
A(5,:)=A(5,:)-A(5,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(4,:)=A(4,:)-A(4,2)*A(2,:)
A(5,:)=A(5,:)-A(5,2)*A(2,:)
A(3,:)=A(3,:)/A(3,3)
A(4,:)=A(4,:)-A(4,3)*A(3,:)
A(5,:)=A(5,:)-A(5,3)*A(3,:)
A(4,:)=A(4,:)/A(4,4)
A(5,:)=A(5,:)-A(5,4)*A(4,:)
disp('solution is')
x= A(:,5)

A =

    12    10    -3    10     0
    -7    -6     7     5     0
     9     9    -5    -1     0
    -4    -3     6     9     0
     8     7    -9    -8     0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0   -0.1667    5.2500   10.8333         0
    9.0000    9.0000   -5.0000   -1.0000         0
   -4.0000   -3.0000    6.0000    9.0000         0
    8.0000    7.0000   -9.0000   -8.0000         0


A =
```

```
    1.0000    0.8333   -0.2500    0.8333         0
         0   -0.1667    5.2500   10.8333         0
         0    1.5000   -2.7500   -8.5000         0
   -4.0000   -3.0000    6.0000    9.0000         0
    8.0000    7.0000   -9.0000   -8.0000         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0   -0.1667    5.2500   10.8333         0
         0    1.5000   -2.7500   -8.5000         0
         0    0.3333    5.0000   12.3333         0
    8.0000    7.0000   -9.0000   -8.0000         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0   -0.1667    5.2500   10.8333         0
         0    1.5000   -2.7500   -8.5000         0
         0    0.3333    5.0000   12.3333         0
         0    0.3333   -7.0000  -14.6667         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
         0    1.5000   -2.7500   -8.5000         0
         0    0.3333    5.0000   12.3333         0
         0    0.3333   -7.0000  -14.6667         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
         0         0   44.5000   89.0000         0
         0    0.3333    5.0000   12.3333         0
         0    0.3333   -7.0000  -14.6667         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
```

```
         0    1.0000  -31.5000  -65.0000         0
         0         0   44.5000   89.0000         0
         0         0   15.5000   34.0000         0
         0    0.3333   -7.0000  -14.6667         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
         0         0   44.5000   89.0000         0
         0         0   15.5000   34.0000         0
         0         0    3.5000    7.0000         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
         0         0    1.0000    2.0000         0
         0         0   15.5000   34.0000         0
         0         0    3.5000    7.0000         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
         0         0    1.0000    2.0000         0
         0         0         0    3.0000         0
         0         0    3.5000    7.0000         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
         0         0    1.0000    2.0000         0
         0         0         0    3.0000         0
         0         0         0    0.0000         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
```

```
         0         0    1.0000    2.0000         0
         0         0         0    1.0000         0
         0         0         0    0.0000         0


A =

    1.0000    0.8333   -0.2500    0.8333         0
         0    1.0000  -31.5000  -65.0000         0
         0         0    1.0000    2.0000         0
         0         0         0    1.0000         0
         0         0         0         0         0

solution is

x =

     0
     0
     0
     0
     0
```

thus verifying the unique solution of the homogeneous equation.

7 Span

4 marks

Suppose that $S = \{v_1, \ldots v_k, v\}$ is a set of vectors in $\Re^n$ and that v is a linear combination of $v_1, \ldots v_k$. If $S' = \{v_1, \ldots v_k, \}$, prove that span $(S) =$ span (S').
Proof: Since S'⊆S, we get span(S')⊆span(S). For the converse inclusion: let $x \in S$; we have

$$x = c_1 v_1 + c_2 v_2 + \ldots + c_k v_k + c_0 v$$

$$v = a_1 v_1 + a_2 v_2 + \ldots a_k v_k$$

$$c_i \in R, i = 0, 1, 2, \ldots, k$$

$$a_i \in R, i = 1, 2, \ldots, k$$

so we have

$$x = (c_1 + c_0 a_1) v_1 + (c_2 + c_0 a_2) v_2 + \ldots + (c_k + c_0 a_k) v_k,$$

i.e. this indicates $x \in S'$, so we get $span(S) \subseteq span(S')$

8 Linear Independence

4 marks

Let $S = \{v_1, \ldots v_k\}$ be a linearly independent set of vectors in $\Re^n$ and let $v \in \Re^n$. Suppose that $v = c_1v_1 + c_2v_2 + \ldots + c_kv_k$ with $c_1 \neq 0$. Prove that $T = \{v, v_2, \ldots v_k,\}$ is a linearly independent set.
Proof. suppose

$$a_1v + a_2v_2 + \ldots + a_kv_k = 0, a_i \in R, i = 1, 2, \ldots, k$$

$$v = c_1v_1 + c_2v_2 + \ldots + c_kv_k$$

so

$$(a_1c_1)v_1 + (a_1c_2 + a_2)v_2 + (a_1c_3 + a_3)v_3 + \ldots + (a_1c_k + a_k)v_k = 0$$

because $S = v_1, v_2, \ldots, v_k$ is linearly independent, we get $a_1c_1 = 0, a_1c_i + a_i = 0, i = 2, 3, \ldots, k$ because $c_1 \neq 0$, we have $a_1 = 0$, then $a_i = 0, i = 2, 3, \ldots, k$ so $T = v, v_2, \ldots, v_k$ is a linearly indepedent set.

9 page 80 #4

2 marks

the augmented matrix is:

```
A=[
1 -3 2 6
0 1 -4 -7
3 -5 -9 -9]
A(1,:)=A(1,:)/A(1,1);
A(2,:)=A(2,:)-A(2,1)*A(1,:)
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(3,:)=A(3,:)/A(3,3)
disp('solution is')
x= A(:,4)
```

the result is as following:

```
A =

     1    -3     2     6
     0     1    -4    -7
     3    -5    -9    -9


A =
```

```
      1    -3     2     6
      0     1    -4    -7
      3    -5    -9    -9


A =

      1    -3     2     6
      0     1    -4    -7
      0     4   -15   -27


A =

      1    -3     2     6
      0     1    -4    -7
      0     4   -15   -27


A =

      1    -3     2     6
      0     1    -4    -7
      0     0     1     1


A =

      1    -3     2     6
      0     1    -4    -7
      0     0     1     1

After back substitution, the solution is

x =

     -5
    -3
      1
```

10 page 80 #8

2 marks

Ax=b, $x \in R^4$, so A has 4 columns and 5 rows.