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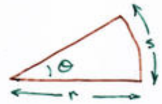
Summary

Rotation, Rolling, Torque & Angular Momentum

Before we can start with rotation we need to examine the relationship between linear and rotational motion. We have



$$s = r\theta$$

which we can differentiate once or twice to give

$$v = r\omega \quad \& \quad a = r\alpha$$

in all of which we have taken r as a constant and, of course, all angles are measured in radians.

$$1 \text{ revolution} = 360° = 2\pi \text{ rad}$$

All three of these rotational quantities can be thought of as vectors with their directions given by the right hand rule.

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11-18 If the angular acceleration and radius are constant then we can divide our linear kinematic equations by r to give

$$\boxed{\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega &= \omega_0 + \alpha t \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0)\end{aligned}}$$

We also note that for circular motion we have

$$\boxed{a_r = v^2/r} \quad \& \quad \boxed{a_t = \alpha r = \frac{d\omega}{dt} r = \frac{d^2\theta}{dt^2} r}$$

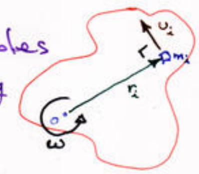
and

$$\boxed{a_{net} = \sqrt{a_r^2 + a_t^2}}$$

and while we're here let's throw in

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T} = \frac{d\theta}{dt}} \quad \& \quad \boxed{v = \frac{2\pi r}{T}}.$$

We use these angular variables because for a rigid rotating body every part of the body has the same $\Delta\theta$, ω & α.



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11-19 The KE of rotation is

$$K = \sum \tfrac{1}{2} m_i v_i^2 = \sum \tfrac{1}{2} m_i (\omega r_i)^2 = \tfrac{1}{2}\omega^2 \sum m_i r_i^2$$

$$\boxed{K = \tfrac{1}{2} I \omega^2}$$

where

$$\boxed{I = \sum m_i r_i^2 = \int r^2\, dm}$$

is the moment of inertia of the rigid body about the specified axis.

The moment of inertia measures the distribution of mass in the rigid body relative to the fixed axis. Just as mass determines the resistance of a body to linear acceleration, the MoI determines the resistance of a rigid body to angular acceleration.

The **Parallel Axis Theorem** can be used to find the MoI of a rigid body about an axis which is a distance D from a parallel axis through the CoM of the body

$$I = I_{com} + MD^2$$

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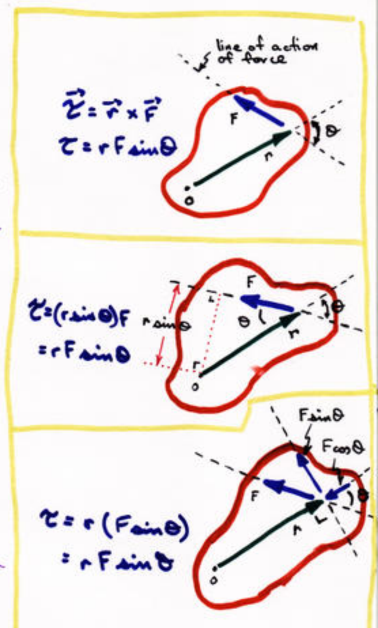
11-20 The torque of a force $\vec{F}$ applied to a rigid body a distance $\vec{r}$ from its pivot point is

$$\boxed{\begin{aligned}\vec{\tau} &= \vec{r} \times \vec{F} \\ \tau &= rF\sin\theta\end{aligned}}$$

where θ is the angle between the $\vec{r}$ & $\vec{F}$ vectors.



The rotational analogue to N2L is

$$\boxed{\sum \vec{\tau} = I\vec{\alpha}}$$

where we must make sure that we have calculated the torques correctly (and also have their sense correct as well).

The WKE theorem for rotation motion is

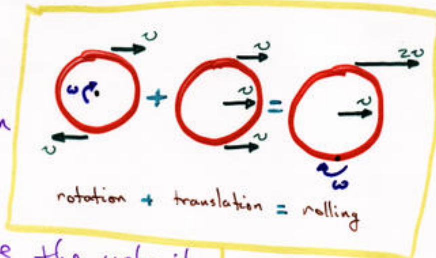
$$W = \Delta K = \tfrac{1}{2} I \omega_f^2 - \tfrac{1}{2} I \omega_i^2$$

and the work done by a torque is

$$\boxed{W = \int_{\theta_i}^{\theta_f} \tau\, d\theta}$$

constant torque ⇒ $\boxed{W = \tau\,\Delta\theta}$ $\boxed{P = \tau\,\omega}$

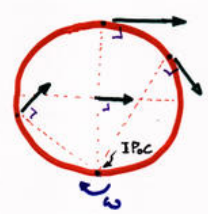
11-21 We can think of rolling as either a combination of rotation and translation or as a rotation about the IPoC.



In the first case the velocity of any point on the object is

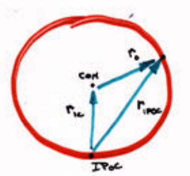
$$\vec{v} = \vec{r}_0 \times \vec{\omega} + \vec{v}_{com}$$

where $\vec{r}_0$ is the position of the point relative to the centre of mass. In the second case the velocity will be



$$\vec{v} = \vec{r}_{IPoC} \times \vec{\omega} = (\vec{r}_0 + \vec{r}_{ic}) \times \vec{\omega}$$
$$= \vec{r}_0 \times \vec{\omega} + \vec{v}_{com}$$

which is the same as above as it must be.



Similarly, we can calculate the KE of rolling in either way

$$K_p = \frac{1}{2} I \omega^2 = \frac{1}{2}(I_{com} + MR^2)\omega^2 = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2}M(R\omega)^2$$
$$= \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2$$

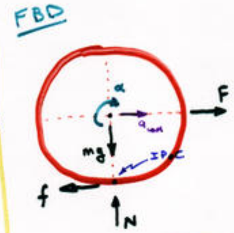
11-22 If the CoM of a rolling object is accelerating then a frictional force must be present to prevent sliding. This friction is static.

When an object is rolling (ie, not sliding) the velocity and acceleration of the CoM are related to the angular velocity and angular acceleration

$\boxed{v_{com} = \omega R}$ & $\boxed{a_{com} = \alpha R}$

where R is the radius of the rolling object.

FBD



N2L

$m a_{com} = F - f$

$I_{com} \alpha = f \cdot R$

solve

$$I_{com}\left(\frac{a_{com}}{R}\right) = (F - m a_{com}) R$$

$$a_{com} = \frac{F}{m + I_{com}/R^2}$$

The angular momentum of an object is defined as

$$\boxed{\vec{\ell} = \vec{r} \times \vec{p} = I\vec{\omega}}$$

and is related to the net torque applied to an object by

$$\boxed{\vec{\tau} = \frac{d\vec{\ell}}{dt}}$$

so if there is no net external torque applied to a system then the angular momentum will be conserved

$$\boxed{\text{no external torque} \Rightarrow \vec{L} = \sum \vec{\ell}_i = \text{constant}}$$

so if a system changes from one configuration to another configuration without an external torque then

$$\vec{L}_i = \vec{L}_f \Rightarrow I_i \omega_i = I_f \omega_f$$