Math 137 Physics Based Section Assignment 1 Due: Sept. 23

NUMBERS:

(Q1) Consider the set of numbers that have the form $\frac{1}{n}$ where n is a natural number of your choosing.
i) Use your calculator to make up a table listing sample inputs and outputs
ii) What happens as n gets large?
iii) Can you think of a logical argument to show that $\frac{1}{n}$ is always bigger than some number L? Also tell me what value you have chosen for L.

(Q2) Repeat Q1 for the set of numbers $\frac{1}{n^2}$. Use the same inputs to construct the new table. What is similar about the two cases? What is different?

(Q3) Now consider the set of numbers of the form $\frac{(-1)^n}{n}$.
i) Show that when n gets large the outputs are very close to zero.
ii) Show that there are two numbers, L_1 and L_2, so that

$$L_1 \leq \frac{(-1)^n}{n} \leq L_2.$$

iii) How does this case differ from the previous two?

(Q4) In the notes we mentioned that division by zero is not defined. By constructing a table, as in the previous questions, show that attempts to find the "limiting behaviour" of

$$\frac{1}{x},$$

where x is a rational number of our choosing, yield different "answers" depending on 'the path we take to get closer to zero'.

(Q5) Consider the sequence of numbers produced by evaluating $\sin(mn)$ with your calculator, where n is a natural number and m is a fixed constant we will specify. Contrast what happens when $m = 1$ with $m = 2\pi$, and $m = \frac{\pi}{6}$. Explain your results clearly.

EQUATIONS

(Q6) Review by solving the following polynomial equations:
i) $x^3 + x^2 = 0$
ii) $x^2 + 6x + 7 = 0$
iii) $x^4 + 6x^3 + 6x^2 - 6x - 7 = 0$ Hint: use part ii)

(Q7) Interpret the following inequalities using the idea of "distance" and solve each:
i) $|x - 4| < 0.1$
ii) $|x^2 - 9| < 0.01$
iii) $|x^3 - 8| < 0.001$

(Q8) How would you use your calculator to find an approximate solution for $\frac{x}{2} = \sin(x)$? Is this an efficient method?

Functions

(Q9) Consider $f(x) = \sin(x^2)$. Write this out in the machine notation used in class so that the output is produced in one step. Now do it for two steps. Use your calculator to show that the two steps cannot have their order switched.

(Q10) Why does the relation $x^2 + y^2 = 2^2$ not define a function?

(Q11) (Not to hand in) The exponential function is one of the basic building blocks of the biological sciences. It is difficult to be precise in defining it mathematically at this point. However:
i) Use your calculator to evaluate e^x for all integers between -5 and 5 $(x = -5, -4, ..., 4, 5)$ and use this to sketch the function.

ii) For each integer compute

$$\frac{e^{N+0.00005} - e^{N-0.00005}}{N + 0.00005 - (N - 0.00005)}.$$

How do these values compare to e^N?

iii) Explain geometrically what I asked you to compute in part ii).

iv) Use your textbook to list some algebraic properties of the exponential. Use them to simplify

$$\frac{e^{2x}}{e^{-3x}} e^{x^2}$$