# Math 138 Physics Based Section
# Assignment 5

**(Q1)** A particle follows each of the following curves. Find the places at which the particle stops moving horizontally, stops moving vertically, and has a minimum speed. Finally sketch the curve.
i) $(x(t), y(t)) = (t^2, t)$ $t > 0$
ii) $(x(t), y(t)) = (t^2, t^2)$ $-10 < t - 10$
iii) $(x(t), y(t)) = (\cos(t), \sin(t))$ $0 \leq t \leq 2\pi$
iv) $(x(t), y(t)) = (\cos(t), \sin(t))$ $0 \leq t \leq 4\pi$
v) $(x(t), y(t)) = (2\cos(t), \sin(t))$ $0 \leq t \leq 2\pi$
vi) For part ii) compute the tangent vector at $t = -2$ and $t = 1$. Add the two tangent lines to your sketch. What happens at $t = 0$?

**(Q2)** Consider the curve $(x(t), y(t)) = (\alpha t + \cos(t), \sin(t))$ $0 \leq t \leq 2\pi$
i) Use Maple or some other means to sketch the curve for three values of $\alpha = 0.5$, $0.9$, $1.1$.
ii) Find the velocity and speed. Sketch the speed as a function of time, again using the $\alpha$ values given above.
iii) What happens at $t = \pi/2$?
iv) HARD: Use parts ii) and iii) to show that when $\alpha > 1$ the curve never crosses itself.

**(Q3)** Consider a bicycle moving with a constant horizontal speed of 2 m/s. If the tires have a radius of 0.5 m, find the equation of a point on the tire. Assume this point is initially found at $(0, 0.5)$.
Hint: after one revolution of the tires the bicycle travels $2\pi r$ meters where $r$ is the radius of the tires.

**(Q4)** Consider the curve $(x(t), y(t)) = \alpha(t^2, t^3)$ $0 \leq t \leq 5$.
i) Write down the arclength integral but do not evaluate it.
ii) Differentiate with respect to $\alpha$ and thus show that the arclength increases as a function of $\alpha$. DO NOT evaluate the integral.

**(Q5)** i) Write down the integral for arclength for the curve in Q2 for a general $\alpha$ and $0 \leq t \leq T$. Do not evaluate.
ii) Use Maple to find the arclength of the curve in Q2 for the three values of $\alpha$ given in Q2i) with $T = 2$.
iii) Make a conjecture of how arclength depends on $\alpha$. Can you come up with a physical reason for your conjecture? You don't need to work out any integrals.

**(Q6)** Find the projection of $\vec{u}$ onto $\vec{v}$ and $\vec{v}$ onto $\vec{u}$ for the following vectors $\vec{u}$ and $\vec{v}$

Recall that the projection of $\vec{u}$ onto $\vec{v}$ is given by

$$\vec{u} \cdot \frac{\vec{v}}{|\vec{v}|}$$

i) $\vec{u} = (3,4)$ $\vec{v} = (3,4)$
ii) $\vec{u} = (1,0)$ $\vec{v} = (3,4)$
iii) $\vec{u} = (0,1)$ $\vec{v} = (3,4)$
iv) $\vec{u} = (3,4)$ $\vec{v} = (0,1)$
v) $\vec{u} = (3,4)$ $\vec{v} = (-4,3)$

**(Q7)** Consider a particle whose position in the plane is given by $\vec{x}(t)$. If $\vec{x}(t) \cdot \vec{x}'(t) = 0$ for all time show that the particle path is a circle centered at the origin. HINT: start with the equation of a circle and take the derivative with respect to time using Chain rule. Once you see how this works, write out $\vec{x}(t) \cdot \vec{x}'(t) = 0$ and work backwards to get the result.

**(Q8)** For each of the following simple harmonic oscillator problems find the frequency $\omega$, write down the general solution $x(t) = A\sin(\omega t) + B\cos(\omega t)$ and use the initial conditions to solve for $A$ and $B$. Recall $v(t) = \frac{d}{dt}x(t)$. Once you know $x(t)$ and $v(t)$ sketch the phase portrait.
i)

$$\frac{d^2}{dt^2}x(t) + 9x(t) = 0$$

$x(0) = 3$, $v(0) = 0$.
ii)

$$\frac{d^2}{dt^2}x(t) + \frac{1}{9}x(t) = 0$$

$x(0) = 3$, $v(0) = 0$.
iii)

$$\frac{d^2}{dt^2}x(t) + 9x(t) = 0$$

$x(0) = 0$, $v(0) = 1$.
iv) In the phase plane description we can write $\vec{p}(t) = (x(t), v(t))$. What two physical quantities does $\frac{d}{dt}\vec{p}(t)$ represent? Does it give two new pieces of information?

**(Q9)** For each combination of force field and particle path compute the work done by the force on the particle during the time interval $0 \le t \le 1$.
i) $\vec{F} = (1,1)$, $(x(t), y(t)) = (t^2, t^3)$
ii) $\vec{F} = (a,b)$, $(x(t), y(t)) = (t^2, t^3)$
iii) $\vec{F} = (x,y)$, $(x(t), y(t)) = (t^2, t^3)$