Math 137 Physics Based Section Assignment 3

(Q1) Consider the three functions $f_1(x) = \sin(\frac{1}{x})$, $f_2(x) = x\sin(\frac{1}{x})$ and $f_3(x) = \frac{\sin(x)}{x}$. Use your calculator to build a table of values for each function as x goes to zero. Discuss the differences between the three cases.

(Q2) Explain why

$$f(x) = \frac{\sin(x)}{x}$$

tends to 0 as $x \longrightarrow \infty$.

(Q3) Consider the two functions $f(x) = x^2$ and $g(x) = 2x - 1$. Using your calculator discuss the similarities and differences *near* $x = 1$.

(Q4)i) For $\epsilon = 0.1$, 0.01 and 0.001 find $\delta > 0$ so that $|x^2 - 1| < \epsilon$ whenever $|x - 1| < \delta$.
ii) For $\epsilon = 0.1$, 0.01 and 0.001 find $\delta > 0$ so that $|x^4 - 1| < \epsilon$ whenever $|x - 1| < \delta$.
iii) For $\epsilon = 0.1$, 0.01 and 0.001 find $\delta > 0$ so that $|x^2 - 100| < \epsilon$ whenever $|x - 10| < \delta$.

(Q5) In each case either evaluate the limit or explain why it does not exist.
i) $\sin x$ as $x \longrightarrow \infty$.
ii) $x^2 + 3x - 1$ as $x \longrightarrow 0$.
iii) $\frac{1}{x}$ as $x \longrightarrow \infty$
iv)

$$\frac{x^2 + 3x}{3x}$$

as $x \longrightarrow \infty$
v)

$$\frac{x^2 + 3x}{3x^2 - 5}$$

as $x \longrightarrow \infty$

(Q6) Use the limit definition of the rate of change, or derivative, to show each of the following:
i) $f'(x) = 3$ for $f(x) = 3x + 5$ (so the derivative agrees with the slope for lines).
ii) $f'(x) = \frac{-1}{x^2}$ for $f(x) = \frac{1}{x}$.
iii) $f'(x) = -\sin x$ for $f(x) = \cos x$ (HINT: use the addition formulae from your textbook or Euler's formula like on assignment 2)

(Q7) Consider $h(x) = f(x)g(x)$. To take the derivative of $h(x)$ we need to take the derivative of a product.

i) Write out the limit definition of the derivative for the product.
ii) Add and subtract $f(x)g(x+h)$ in the numerator and regroup appropriately.
iii) Use the fact that the limit of a sum can be split up into the sum of the limit (provided each limit exists on its own) to split up the result into two limits.
iv) Evaluate the two limits to yield the **PRODUCT RULE FOR DERIVATIVES**, namely

$$\frac{df(x)g(x)}{dx} = f'(x)g(x) + f(x)g'(x)$$

(Q8) On a computer we use approximations of limits to compute derivatives. Fix $\Delta t = 0.01$ and the two functions $a(t) = 3t + 5$ and $b(t) = t^2$. Using your calculator find the error that the approximation

$$\frac{a(t+\Delta t) - a(t)}{\Delta t}$$

and

$$\frac{b(t+\Delta t) - b(t)}{\Delta t}$$

to the actual derivative makes at $t = 1$, $t = 10$ and $t = 100$. Briefly explain your results.