Math 137 Physics Based Section Assignment 4

(Q1) Consider a point particle of mass m moving under the influence of a force $f(t)$ given below. For each $f(t)$, time interval and initial conditions given below, find $v(t)$ by reversing the derivative machine. By the same means find $x(t)$. Finally sketch $x(t)$, $v(t)$ and $f(t)$.
i) $f(t) = \sqrt{t}$ for $0 < t < 2$ and $(x(0), v(0)) = (0, 0)$
ii) $f(t) = 0$ for $0 < t < 2$ and $(x(0), v(0)) = (0, 5)$
iii) $f(t) = 0$ for $0 < t < 5$ and $(x(0), v(0)) = (5, 0)$
iv) $f(t) = t - t^2$ for $0 < t < 1$ and $(x(0), v(0)) = (0, 0)$
v) $f(t) = t - t^2$ for $0 < t < 2$ and $(x(0), v(0)) = (0, 0)$

(Q2) Consider the point particle problem as given in (Q1). Find a so that if $x(0) = v(0) = 0$ and $f(t) = t - at^2$ then $x(2) = 0$.

(Q3) Use Euler's method with $\Delta t = 0.1$ to approximate the solution to (Q1) part iv). Include a table of your calculations in your answer and discuss the error in the approximate solution.

(Q4) Now consider point particle problems in which the applied force can be controlled and hence changed either in form or value instantaneously (an electromagnet could be used to do this, for example). Since these problems cannot be solved by the straightforward reversal of the derivative machine, split each problem up into two or more parts like those solved in (Q1). Assuming $x(0) = v(0) = 0$, solve each problem for $v(t)$ and subsequently $x(t)$ and sketch $x(t)$, $v(t)$ and $f(t)$.
i) $f(t) = 1$ for $0 < t \le 1$ and $f(t) = 0$ for $1 < t \le 2$
ii) $f(t) = t$ for $0 < t \le 1$ and $f(t) = 1 - t$ for $1 < t \le 2$
iii) $f(t) = t$ for $0 < t \le 1$, $f(t) = 0$ for $1 < t \le 2$ and $f(t) = -t^2$ for $2 < t \le 3$

(Q5) Consider a projectile fired with a speed v_0 at an angle θ from the horizontal.
i) Sketch the situation and choose your axes so that $(x(0), y(0)) = (0, 0)$
ii) Write out the two components of the velocity as functions of θ and v_0 iii) Write out Newton's Second Law for each component along with the appropriate initial conditions for each component.
iv) For the 6 cases that result from the combinations of $v_0 = (1, 5, 10)$ and $\theta = (\pi/6, \pi/4, \pi/3)$ solve the initial problems and thereby find the distance along the x-axis travelled when the projectile hits the ground.
v) Sketch $x(t)$ and $y(t)$ for $v_0 = 1$ and $\theta = (\pi/6, \pi/4, \pi/3)$.

(Q6) Find the rates of change, or derivatives, of:
i) $a(t) = \sin(t^2 + 2t)$

ii) $a(t) = \sqrt{(t^3 + 1)}$
iii) $a(t) = \frac{1}{(t^2+1)}$
iv) $a(t) = (t + 1)^2$ Using the CHAIN RULE!

(Q7) In a two step industrial process the concentration of the chemical produced is given as a function of the concentration of the chemical produced in the first step of the industrial process. However, the concentration of the first chemical cannot be measured directly, and is known instead as a function of the heat input into the endothermic reaction. Defining all terms, write the concentration of the product chemical as a function of time, using several steps. Use the Chain Rule to find the rate of change of the concentration of the product with time.