Math 138 Physics Based Section
Assignment 8

(Q1) Find the Maclaurin series of the following functions and find the radius of convergence
i) $f(x) = (2+x)^5$
ii) $f(x) = \sin(2x)$
iii) $f(x) = \exp(-3x)$
iv) $f(x) = \cos(x^4)$
v) $f(x) = \ln(1+x)$ HINT:

$$\ln(1+x) = \int_0^x \frac{1}{1+t} dt$$

vi) $f(x) = \frac{1}{2-x}$
vii) $f(x) = \frac{2+x}{1-x}$
viii) $f(x) = \frac{5}{1-x-6x^2}$ HINT: example 9 on page 111 is similar.

(Q2)i) Use the Maclaurin series to find

$$\int \exp(-x^2) dx$$

ii) Use part i) along with Maple to estimate

$$\int_0^1 \exp(-x^2) dx.$$

What technique did you use to make sure your estimate was good?

(Q3) Use the Mclaurin series to evaluate the following limits as $x \longrightarrow 0$:
i)

$$\frac{\exp(x) - 1 - x}{x^2}$$

ii)

$$\frac{\cos(x) - 1 - x}{x}$$

iii)

$$\frac{\cos(x) - 1 - x^2}{5x^4}$$

(Q4) Consider the simple harmonic oscillator equation

$$\frac{d^2x}{dt^2} + \omega^2 x(t) = 0$$

with the initial conditions $x(0) = 1$, $v(0) = x'(0) = 0$. i) Solve the initial value problem.
ii) If ω is very small approximate the solution you have found.
iii) Recover your approximation to leading order WITHOUT solving the DE first. HINT: write $x(t) = x_0(t) + \omega^2 x_1(t) + ...$ where $\omega^2 \ll 1$ substitute this into the equation and find the biggest piece.
iv) Does this make physical sense? Explain using the example of a mass on a spring for which

$$\omega^2 = \frac{k}{m}$$

(Q5) The equation for a pendulum is written as

$$\frac{d^2\theta}{dt^2} + \omega^2 \sin(\theta(t)) = 0.$$

This is not the simple harmonic oscillator equation

$$\frac{d^2x}{dt^2} + \omega^2 x(t) = 0.$$

If $\theta(t)$ is small use a Maclaurin series to derive the SHO from the pendulum equation. What would an "improved" approximation look like? Could you solve it?

(Q6) Let's reconsider one of the DEs we have seen before on assignments and tests:

$$\frac{dy}{dt} = y(2-y)$$

Recall that there are two fixed points $y = 0$ and $y = 2$.
i) Let $y = 0 + \epsilon y_p$, or in other words start at the fixed point and assume you have moved slightly away. Find the leading order approximate DE for y_p and solve it. What does it tell you about the long time behaviour of the solutions to the original equation?
ii) Repeat part i for $y = 2 + \epsilon y_q$.

(Q7) Find the first three non-zero terms of the Taylor series for $\sin(x)$ about the following points:
i) $a = 0$
ii) $a = \pi$
iii) $a = \pi/2$
iv) $a = \pi/4$
v) $a = \pi/6$

(Q8) Consider the derivation of the Maclaurin series of $\arctan(x)$ we did in class.
i) For an expansion to 5 terms what is the EXACT expression for the error?
ii) What is the UPPER BOUND on the error?
iii) Use the upper bound on error to estimate $\arctan(0.9)$ with $N = 5$, 50 and 500 terms.
iv) Use the upper bound on error to find N so that the error in the approximation of $\arctan(0.9)$ is less than 0.1 and 0.001.
v) Could you repeat parts iii and iv for $x = 1.1$? Explain.