Math 137 Physics Based Section
Assignment 7

(Q1)i) A rectangular enclosure is to be built against the side of a barn that is 40 meters long. If the total amount of fencing available is 200 meters find the dimensions of the enclosure that will yield the largest area. Note, the enclosure cannot extend past the end of the barn wall, but does not have to cover the entire wall.
ii) Solve the above problem if only 100 meters of fencing is available.

(Q2) A factory can produce one of two products A and B. Product A can be produced at a rate of 10 a day and yields a profit of 5 dollars per product. Product B can be produced faster, namely 15 a day, however it yields smaller profit of 4 dollars per product. A manager is making plans for the next month (30 days) and wishes (of course) to maximize profit.
i) By defining an appropriate input variable find an equation in one variable for the profit the factory makes.
ii) Find the maximum profit and the amount of each product produced to earn this profit.
iii) What is the 'hidden' assumption you are making and why would it not apply in a real factory?

(Q3) Consider the so-called Lenard-Jones potential given, for our discussion, by

$$U(r) = \frac{1}{r^{12}} - \frac{1}{r^6}$$

i) Plot $U(r)$ for $0.9 \leq r \leq 2$.
ii) On the same interval find the r values at which $U(r)$ is minimized and maximized.
iii) In physics, if a potential is known then the force due to that potential is given by $F(r) = -U'(r)$. Find $F(r)$ and interpret the minimum of $U(r)$ you found in the last problem physically.

(Q4)i) Find the minimum distance between the point $(2, 3)$ and the graph of the function $f(x) = x - 7$.
ii) Find the minimum distance between the point $(2, 3)$ and the graph of the function $f(x) = 6x - 9$.
iii) Find the minimum distance between the point $(2, 3)$ and the unit circle centered at the origin.

(Q5)i) For the function $f(x) = x^3 - x$ on the interval $[-1, 1]$ show that the hypotheses of the Mean Value Theorem are satisfied and thus that there is at least one c, $-1 < c < 1$ at which $f'(c) = 0$.
ii) Is there only one c? Does Mean Value Theorem tell you whether there is more than one? Explain.

iii) Consider $f(x) = x$ when $x > 0$ and $f(0) = 5$. Show that even though $f(5) - f(0) = 0$, there is no x, $0 < x < 5$ at which $f'(c) = 0$. Does this result contradict the Mean Value Theorem?

(Q6)i) Consider the interval $[-1, 1]$. If $f'(x) > 0$ for all $-1 < x < 1$ and $f(.)$ is continuous on $[-1, 1]$ use the Mean Value Theorem to prove that $f(1) > f(-1)$
ii) BONUS extend your proof for any $-1 < a < b < 1$.

(Q7) A police officer stops a car on highway 401 and gives the driver a speeding ticket. The driver claims that he was driving well below the speed limit and asks to see the radar gun, to which the officer responds by saying he doesn't need one. He tells the driver, you left Toronto, 100 kilometers away and got here in 45 minutes. The speed limit is 100 kilometers per hour, so I know that somewhere along the way you were speeding. Use the Mean Value Theorem to explain why the police officer is right.

(Q8) The linear approximation of a function $f(x)$ at a point $x = x_0$ can be written in differential form as

$$\Delta f(x_0) = f'(x)\Delta x$$

i) By defining terms appropriately derive the above from our past work on the linear approximation.
The above notation can be used to get quick estimates of how a change in the input affects the output.
ii) Consider the volume of a sphere

$$V(r) = \frac{4}{3}\pi r^3.$$

Use the differential form of the linear approximation to compute the effect on the volume of an increase of 1 centimeter from an initial radius of 1 meter. How does the estimate compare with the actual change?
iii) Repeat part ii) for a decrease of 1 centimeter. iv) Repeat part ii) for an increase of 10 centimeters.