Math 137 Physics Based Section
Assignment 6

(Q1) Find the rates of change of the following:
i) $f(x) = \exp(-x^2)$
ii) $f(x) = x\exp(-x^2)$
iii) $f(x) = \ln(\frac{1+x}{1+x^2})$
iv) $f(x) = 2^x$ (use both exp and ln to rewrite and then differentiate)
v) $f(x) = 2^{(-x^2)}$
vi) $f(x) = x\ln(x) - x$

(Q2) Given the equation

$$y^2 + \exp(-x) + 1 = 0$$

which defines the function $y(x)$ implicitly, use the derivative machine to find

$$\frac{dy}{dx} = \frac{\exp(-x)}{2y}.$$

However this result is meaningless. Explain why. (HINT: to have meaning $y(x)$ must output real numbers)

(Q3) Sketch the following (using derivatives may prove helpful) and be sure to specify the domain of valid inputs.
i) $f(x) = \exp(-x^2)$
ii) $f(x) = x\exp(-x)$
iii) $f(x) = 2^x$
iv) $f(x) = 2^{(-x^2)}$
v) $f(x) = x\ln(x) - x$

(Q4) Consider the function $f(x) = x^2$.
i) Find the tangent line at $x = 0$. What does this mean for the linear approximation?
ii) Find the set of x for which the error made by the linear approximation is less than 0.01
iii) Now consider the function $g(x) = x^4$. Repeat parts i) and ii) for this function. iv) Let's say we wanted to extend the idea of the linear approximation near $x = 0$ by saying the quadratic approximation is given by

$$Q(x) = f(0) + f'(0)x^2 + \frac{1}{2}f''(0)x^2$$

Explain why we need the $\frac{1}{2}$ in front of the quadratic term, find $Q(x)$ for $g(x) = x^4$ and explain your result.

(Q5) Consider the function $f(x) = x \exp(-x)$ that you sketched above.
i) Using the first and second derivative test find the local and global maxima and minima on the interval $0 \leq x \leq 2$.
ii) How would the result change if the interval was instead $0 \leq x < 2$?
iii) How would the result change if the interval was instead $0 < x < 2$?

(Q6) It is a mathematical theorem that any continuous function on a closed interval (written $[a, b]$ meaning $a \leq x \leq b$) attains its maximum and minimum.
i) On the open interval (written (a, b) or $a < x < b$) the result no longer holds. Give an example illustrating this fact and explain why no maximum or minimum is reached.
ii) Even if we maintain that the interval is closed a discontinuous function needs not attain its maximum or minimum. By defining a function in pieces (something like $f(x) = 1$ when $x < 0.5$ and $f(x) = 2$ when $x \geq 0.5$) find an example of a function on $[0, 1]$ which does not achieve its maximum.

(Q7) Use what you know about logarithms and exponentials, along with the Chain Rule, to solve the differential equation

$$\frac{dx}{dt} = x(1 - t^2)$$

where $x(0) = 5$. What happens to $x(t)$ as t gets large?