Math 138 Physics Based Section Assignment 1

(Q1) Evaluate the following integrals:
i) $\int(\frac{1}{\sqrt{x}}\cos(\sqrt{x}))dx$
ii) $\int_1^2(\frac{1}{\sqrt{x}}\cos(\sqrt{x}))dx$
iii)$\int\frac{t}{1+t^2}dt$
iv) $\int\frac{t}{1+t^4}dt$
v) $\int\frac{7t-3}{t(t-1)}dt$. For what values of t is the answer valid
vi) $\int\frac{1-3t}{t^2}dt$
vii) $\int\frac{5t^2+2}{t^4+2t^2}dt$
viii) $\int t\sin tdt$
ix) $\int_2^4 m\sin(m)dm$
x) $\int_0^5\exp(-t)\sin(t)dt$

(Q2)i) Find the area beneath the graph of $g(x)=x^2$ and above the horizontal line $y=-1$ for $0\leq x\leq 5$
ii) Find the area to the left of the curve $g(x)=x^2$ and to the right of the y-axis for $0\leq x\leq 2$.
iii) Find the area below the graph of $f(x)=\sin(\pi x)$ and above the x-axis for $0\leq x\leq 200000$.
iv) Use your answer in part ii) to find the volume of the same region rotated about the y-axis. Include a clear sketch of the Riemann sums.

(Q3)i) Define

$$f(x)=\int_1^x\exp(-t)dt$$

Find $f'(x)$. Did you need to do any integration?
ii) Repeat i) for

$$f(x)=\int_1^x\exp(-t^2)dt$$

Hint: You can't do the integral.
iii) If

$$f(x)=\int_1^x g(t)dt$$

find $f'(x)$ in terms of $g(.)$.
iv) Repeat the above for

$$f(x)=\int_1^{x^2}g(t)dt$$

find $f'(x)$ in terms of $g(.)$.
v) Now let's say we have an x inside the integral. Say

$$f(x) = \int_1^2 (x^2 t + \sin(xt))dt$$

find $f'(x)$ as an integral (DO NOT evaluate the integral).

(Q4) Despite its apparent simplicity and usefulness (in statistics for example) the improper integral

$$\int_0^\infty \exp(-x^2)dx$$

is not possible to carry out using elementary functions (something worth thinking about, actually). Nevertheless we could at least try to get some estimates:
i) Using five boxes which split the interval $[0, 4]$ into equal width boxes, approximate the above integral. By making an appropriate choice can you make sure that your estimate is a lower bound?

ii) Since we are ignoring the entire set of inputs $x > 4$ we cannot make our approximation an upper bound with our existing definition of integral. You might guess that by allowing yourself to treat the region $x > 4$ as a single box you could find an upper bound for the Riemann sum. Show that this approach does not work.

(Q5)i) Show that the function

$$g(x) = \int_{-10}^{x} |t|dt$$

has a valid derivative for all $x > -10$. Explain why this is possible with reference to the fact that $|x|$ does not have a derivative at $x = 0$.
ii) Find $\int |x|dx$ for $x \geq 0$ and for $x < 0$. iii) Set the arbitrary constants in part ii) to zero and sketch the resulting function on $-2 \leq x \leq 2$. On your sketch also include $|x|$.

(Q6) Consider the following functions with potential jump discontinuities.

$$f(t) = t + t^2 H(t-2),$$

$$g(t) = t + (t^2 - 4)H(t-2),$$

$$p(t) = \sin(2\pi t) + H(t - 0.75)(\exp(-t + 0.75) - 1).$$

i) Sketch all three functions and identify any points of discontinuity.
ii) Find

$$\int_0^4 f(t)dt$$

(Q7) Recall that integrals whose domain went out to infinity required that the function being integrated goes to zero fast enough (for example $f(x) = 1/x$ does not go to zero fast enough). We now consider functions which "blow up" at some point. To be concrete we consider $g(x) = x^{-\alpha}$ for $\alpha > 0$ near the point $x = 0$. The integral in question will be

$$\int_0^1 g(x)dx$$

i) Define the above improper integral using a limit (your notes should be helpful).
ii) Show what happens when $\alpha = 1/2$, 1 and 2.
iii) Based on ii) make a conjecture on the general result, and provide an explanation why you guessed what you did.
iv) By defining a new variable $y(x)$ show that you can convert the above case of improper integrals to the case discussed in your notes.
v) Based on your above work define the two-side improper integral

$$\int_{-1}^1 g(x)dx$$