Math 138 Physics Based Section
Assignment 3

Q1 Consider the very simplest Simple Harmonic Oscillator equation

$$\frac{d^2x}{dt^2} + x = 0$$

i) Use the Laplace transform to solve the initial value problem with $x(0) = 1$ and $x'(0) = 0$

ii) using part i) show that the initial value problem $x(0) = x_0$ and $x'(0) = 0$ has the solution

$$x(t) = x_0 \cos(t))$$

iii) Show, using Laplace transforms, that if we change the governing equation to

$$\frac{d^2x}{dt^2} + 4x = 0$$

then the solution to the initial value problem is $x(0) = 1$, $x'(0) = 0$ is $x(t) = \cos(2t)$.
iv) Use Laplace transforms to solve the initial value problem $x(0) = 0$ and $x'(0) = 1$. For the governing equation in part iii)
v) Explain the factor of 1/2 you found in part iv)

vi) If

$$x'' + x = \sin(t)$$

solve the initial value problem with $x(0) = 0$ and $x'(0) = 1$.
vii) In the previous solution describe the dominant behaviour for large t.

(Q2) Consider the damped oscillator equation

$$x''(t) + bx'(t) + x(t) = 0$$

where $b > 0$ ON PHYSICAL GROUNDS.
i) Let $x(t) = \exp(at)$ and show that a satisfies the quadratic equation

$$a^2 + ba + 1 = 0$$

ii) If $b^2 > 4$ show that a has two solutions.
iii) For what values of b can we get a positive solution for a?
iv) Interpret the result from part iii).
v) If $b = \sqrt{3}$ find a

vi) Show that in this case $x(t) = \exp(-\sqrt{3}t/2)\sin(\frac{1}{2}t)$ is a possible solution.

Q3i) Without solving find one interesting fact about the solution to the initial value problem

$$x'(t) = \tan(t)$$

where $x(0) = 2$.

ii) Without solving, show that the solution to the initial value problem

$$x'(t) = x^2(10 - x)$$

where $x(0) = 2$ remains bounded for all $t > 0$
iii) For the problem in part ii) what happens for large times?

iv) If $x'(t) < 0$ for all t does $x \longrightarrow -\infty$ as $t \longrightarrow \infty$?

v) If $x \longrightarrow -\infty$ as $t \longrightarrow \infty$ does $x'(t) < 0$ for all t?

Q4 Consider a cylindrical tube with a radius of 2 meters. The tubing is rusting and thus the concentration of iron in the tube is given by

$$C(r) = C_0(0.5 + r/2)$$

i) What are the appropriate units for the reference concentration C_0? Why write the formula the way we did?

ii) If we now make things easy and set $C(r) = 0.5 + r/2$ use the cylindrical shells technique to find the total amount of iron in a 3 meter section of pipe.

iii) What is the average concentration in the pipe?

iv) If $C(r) = C_0 + \sin(\pi r)$ explain WITHOUT calculating why the average of $C(r)$ is not merely C_0. Could you change $C(r)$ to get the right result?

Q5i) Consider the equation that governs the decaying exponential $y(t) = C_0 \exp(-2t)$:

$$y'(t) + 2y = 0$$

Use the trick for separable equations to find the solution and thus explain why the arbitrary constant is multiplied and not added.
ii) Solve the separable DE

$$y'(t) - \frac{y}{t} = 0$$

and sketch some solution curves.

iii) Does the initial value problem $x(0) = 3$ for the previous equation have a solution?

iv) Sketch the direction fields for the DEs in parts i) and ii).

(Q6)i) Consider a skydiver falling under the influence of gravity and a drag force whose magnitude is given by $kv(t)$ and which acts AGAINST the direction of propagation.
i) Formulate the problem using Newton's Law for $v(t)$ NOT $x(t)$.
ii) Show that if there is no acceleration then $v = v_{terminal} > 0$.
iii) Discuss how terminal velocity depends on the various physical parameters in the problem.
iv) Solve the initial value problem with $v(0) = 0$ and show that $v(t)$ tends to the terminal velocity.