Math 137 Physics Based Section Assignment 2 Due: Sept. 30

(Q1) Consider the Universal Law of Gravitation from the notes. Is it valid for all inputs (distances)? If the numerator is chosen to equal 1 as in the notes, for what value of r will the resulting force be less than 10^{-6}. What does this say about gravity?

(Q2) The idea of building functions may seem complicated, so why not try something easier, namely building numbers.
i) Show that

$$\sum_{k=1}^{k=\infty} \frac{1}{2^k} = 1.$$

Note that this is a geometric series and hence you can use a formula.
ii) Use your calculator to evaluate the series for the first 3, 5 and 10 terms. How many terms are needed to get closer than 0.01 to 1.

(Q3) Much like the rational numbers are two integers divided by one another, rational functions are two polynomials divided by one another. Consider

$$f(x) = \frac{x^2 - 1}{x + 1}$$

and

$$g(x) = \frac{3x^2 + 4x + 1}{x^2 - 5x + 6}.$$

i) For each function give a valid set of inputs. ii) Sketch each function. iii) Use your calculator to determine what are typical outputs of $g(x)$ when x is large. How is this reflected in the formula?

(Q4) Sketch two periods of each function and carefully label the graph:
i) $f(x) = \sin(x)$
ii) $f(x) = \sin(2x)$
iii) $f(x) = \sin(\frac{2\pi x}{5})$
iv) $f(x) = \sin(\frac{2\pi x}{5} + \frac{\pi}{2})$

(Q5) Use the input-output style of argument from the notes to argue that $f(x) = \sin(x + \frac{\pi}{2})$ and $g(x) = \cos(x)$ are actually the same function. Now provide a geometrical interpretation using the circle definition of sine and cosine.

(Q6) There are many identities involving sine and cosine. If one uses the remarkable Euler's formula, many are simple to prove. Euler's formula reads

$$\exp(ix) = \cos(x) + i\sin(x)$$

where i is the imaginary number representing $\sqrt{-1}$. If we just agree to use i as a place holder along with the rule $i^2 = -1$, then $(a+ib)^2 = a^2 + i^2b^2 + 2abi = a^2 - b^2 + i2ab$. Use the properties of the exponential (use your text if you wish) to show

$$\cos(2x) = \cos(x)^2 - \sin(x)^2$$

and

$$\sin(2x) = 2\sin(x)\cos(x).$$

Hint: If $a+ib = c+id$ then not only does $a=c$, but $b=d$ as well.

(Q7)i) The function $f(x) = \tan(x)$ is invertible provided its domain (input) is restricted. Find the set of inputs that includes $x=0$ for which $f(x)$ is invertible and sketch both $f(x)$ and its inverse.
ii) Repeat i) for the set of inputs that includes $x = -\pi$.

(Q8) Sketch $f(x) = 3 - |x+1|$, $g(x) = |x^2 - 3|$ and $h(x) = |x^3 - 8|$ for $-3 \leq x \leq 3$.