P122 Web Copy

Physics 121 Final Exam Fall 2004 With Sample Solutions And Marking Schemes

UNIVERSITY OF WATERLOO
DEPARTMENT OF PHYSICS

PHYSICS 121 - FINAL EXAMINATION

DATE: **Monday 20 December, 2004** **TIME: 9:00 AM - 12:00 Noon (3 Hours)**

SURNAME: ______________________
(Please Print)

Given Name(s) ______________________
(Please Print)

I.D.No.: ______________________

SIGNATURE ______________________

Circle Your Professor's Name and Your Lecture Section No.

001 O'Donovan TR 8:30
002 O'Donovan MWF 9:30
003 Brandon MWF 11:30

LECTURE SECTON NO.: ______________

Type of Exam: **Closed Book**

Aids Permitted: Calculator
Writing Implements

Instructions: Answer all eight questions. Each numbered question is of equal value.

Where boxes are provided, write your answers with the appropriate units in those boxes.

If you need more space, use the back of the previous page or the blank page at the end, but leave a note indicating where any question has been continued.

To earn full marks, your solutions must show your reasoning.

Assume all given numerical data are accurate to three significant figures.

Useful Data:

$g = 9.80\ m/s^2$

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

1.(a) A football player kicks a ball from ground level toward a goal post which is 40.0 m away. The kicker gives the ball a speed of 25.0 m/s at an angle of 35.0° above the horizontal ground. If the goal post has a cross-bar of height h = 3.00 m above the ground, determine by how much the ball clears the cross-bar.

Projectile Motion: Use $x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$

$40 = 0 + (25\cos 35°)\,t^* + 0$

$t^* = 40/25\cos 35° = 1.953\ s$

$y(t) = 0 + (25\sin 35°)\,t + \frac{1}{2}(-9.80)\,t^2$

$y(t^*) = 0 + (25\sin 35°)(1.953) - 4.90(1.953)^2$

$= 28.01 - 18.69 = 9.314\ m \qquad \Delta h = 9.314 - 3.00 = 6.31\ m$

Δh = 6.31 m　(5)

(b) Determine the velocity vector of the ball at the instant it crosses the goal line. Express your answer in terms of a speed and an angle in degrees up or down (circle one) with respect to the horizontal.

$v_x = v_{x_i} + a_x t = 25\cos 35° + 0 = 20.48$

$v_y = v_{y_i} + a_y t = 25\sin 35° - 9.80(1.953)$

$= 14.34 - 19.14 = -4.800$

$v = \sqrt{(20.48)^2 + (-4.800)^2} = 21.00\ m/s$

$\tan\phi = v_y/v_x = \frac{-4.80}{20.48} = -0.2344 \Rightarrow \phi = 13.2°$

down because v_y is $-$ve.

v = 21.0 m/s　(3)

φ = 13.2° up / down　(2)

(c) Determine the maximum height reached by the football.

Use $v_y^2 = v_{y_i}^2 + 2a_y(y - y_i)$

At highest point, $0 = (25\sin 35°)^2 + 2(-9.80)(h_m - 0)$

$h_{max} = \frac{(14.34)^2}{19.6} = \frac{205.6}{19.6} = 10.5\ m$

Alt Method: Find time to top using $v_y = v_{iy} + at$

and substitute this time into $y = y_i + v_{y_i}t + \frac{1}{2}a_y t^2$

h_{max} = 10.5 m　(4)

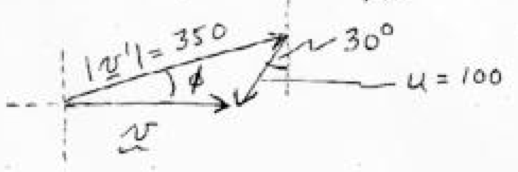
(d) The pilot of an airplane wishes to fly due east after leaving the Waterloo-Wellington airport on a windy day. Her airplane has an air speed of 350 km/h, but the wind is blowing at 100 km/h in a direction 30.0° west of south. What is the ground speed of the airplane as the pilot flies it due east?

v = 289 km/h　(6)

Use the relative velocity idea

$\underset{\text{wrt } E}{\underset{\sim}{v}_P} = \underset{\text{wrt Air}}{\underset{\sim}{v}'_P} + \underset{\text{wrt } E}{\underset{\sim}{u}_{Air}}$

P ⇒ Plain
E ⇒ Earth
Air ⇒ Air



} Construct velocity vector triangle with properties as stated.

$|\underset{\sim}{v}| = 350\cos\phi - 100\sin 30°$

But $\sin\phi = 100\cos 30°/350 = 0.8475 \Rightarrow \phi = 14.33°$

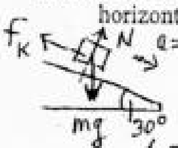
$|\underset{\sim}{v}| = 350\cos 14.33° - 100\sin 30°$

$= 339.1 - 50.0$

$= 289\ km/h$

Total 20

2.(a) A 5.00 kg block slides with constant speed down a plane inclined at an angle of 30.0° to the horizontal. What is the coefficient of kinetic friction between the block and the plane?



$\Sigma F_x = ma_x \Rightarrow$

$mg\sin 30° - f_K = m(0)$ where $f_K = \mu_K N$

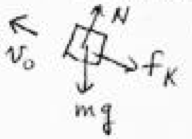
$\Sigma F_y = ma_y \Rightarrow N - mg\cos 30° = m(0) \quad \therefore N = mg\cos 30°$

$f_K = \mu_K N = \mu_K mg\cos\theta$ and $mg\sin 30° = \mu_K mg\cos 30°$

$\therefore \mu_K = \frac{mg\sin 30}{mg\cos 30} = \tan 30° = 0.577$

$\boxed{\mu_K = 0.577}$ (4)

(b) After the block reaches the bottom of the plane it is projected back up the incline with an initial speed of 4.00 m/s. What distance, d, up the inclined plane will the block move before coming to rest?



Choosing +ve direction UP the incline,

$-f_K - mg\sin 30° = ma$

$a = -\mu_K g\cos 30° - g\sin 30° = -0.577(9.80)\cos 30° - 9.80\sin 30°$

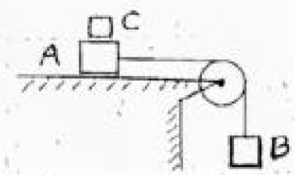
$= -4.897 - 4.90 = -9.80$ (ie DOWN the incline)

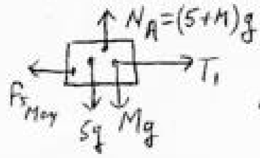
Use $v^2 = v_i^2 + 2a(x - x_i)$

$0 = (4.00)^2 + 2(-9.80)(d - 0) \Rightarrow d = 16.0/19.6 = 0.816\,m$

$\boxed{d = 0.816 \text{ m}}$ (5)

(c) Three blocks are arranged at rest as shown, (M_A = 5.00 kg and M_B = 2.75 kg), connected by a light cord over a frictionless pulley with moment of inertia 0.225 kg m² and radius 0.150 m. The coefficients of friction between block A and the table are μ_S = 0.250 and μ_K = 0.200. Find the minimum mass of block C such that A does not slide.





$fs_{Max} = \mu_s N_A = 0.250(5 + M_m)g$

For $a = 0$ $\quad T_1 = T_2 = m_B g = 2.75g$

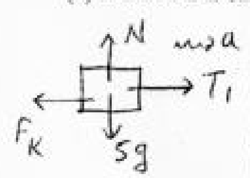
and $fs_{Max} = T_1$

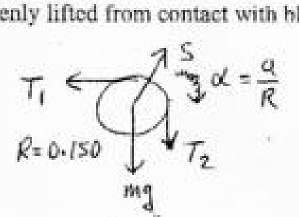
$\boxed{M_{min} = 6.00 \text{ kg}}$ (5)

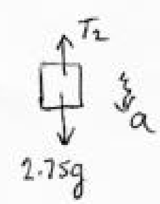
$\therefore \; 2.75g = \mu_s(5 + M_m)g$

$M_{min} = -5 + \frac{2.75g}{\mu_s g} = -5 + \frac{2.75}{0.250} = -5 + 11 = +6.00\,kg$

(d) If block C is suddenly lifted from contact with block A, find the acceleration of block A.







$\boxed{a_A = 0.966 \text{ m/s}^2}$ (6)

$\Sigma F = ma$

$T_1 - \mu_K N = m_A a$

$\underbrace{T_1 - (0.2)(5)(9.80) = 5a}_{(+)}$

$\Sigma\tau = I\alpha$

$RT_2 - RT_1 = I\alpha$

$\alpha = \frac{a}{R}$, $I = 0.225$

$\underbrace{T_2 - T_1 = \frac{I}{R}\frac{a}{R} = 10.0a}_{(+)}$

$\Sigma F = ma$

$2.75g - T_2 = m_B a$

$\underbrace{2.75g - T_2 = 2.75a}_{(+)}$

(+) Add 3 eqns to eliminate T_1 and T_2:

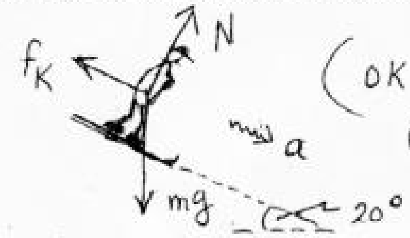
$-9.80 + 26.95 = 5a + 10a + 2.75a$

$a = \frac{17.15}{17.75} = 0.966\ m/s^2$ [Alt. answer: $0.0986g$]

Total 20

3. A downhill ski racer (KS) of mass 60.0 kg travels a distance of 50.0 m down an inclined plane which makes an angle of 20.0° to the horizontal, and which has $\mu_K = 0.150$. Neglect any air resistance.

(a) Draw a free body diagram of the skier, showing all the forces on her.



(OK to show mg force in Component form)

③

(b) Determine the magnitude of each of your forces shown in part (a) and find the constant acceleration of KS.

a = 1.97 m/s²

⑤

$mg = (60.0)(9.80) = \underline{588.0 N}$

$N = mg\cos 20° = (60.0)(9.80)(0.93969) = \underline{552.5 N}$

$f_K = \mu_K N = (0.150)(552.5) = \underline{82.88 N}$

$mg\sin 20° - f_K = ma$

$(588.0)(0.3420) - 82.88 = 60.0a \Rightarrow a = \frac{118.2}{60.0} = \underline{1.97\ m/s^2}$

(c) Calculate the work done on KS by each of the forces that you have shown in part (a) during the motion described.

$W_{mg} = mgd\cos 70° = (60.0)(9.80)(50.0)(0.34202) = 10{,}055\ J$ ie $\underline{10.1\ kJ}$

$W_N = Nd\cos 90° = \underline{0\ J}$

④

$W_{f_K} = f_K d\cos 180° = (82.88)(50.0)(-1) = -4144\ J$ ie $\underline{-4.14\ kJ}$

(d) Using your answer for the acceleration from part (b), find the velocity of KS at the bottom of the 50.0 m section of the incline if she started at the top with an initial velocity of 5.00 m/s down the incline.

v_f = 14.9 m/s

④

$a = 1.97\ m/s^2$ down the incline (+x)

Use $v_f^2 = v_i^2 + 2a(x - x_i)$ since acc'n is constant.

$v_f^2 = (5.00)^2 + 2(1.97)(50.0 - 0)$

$= 25.0 + 197.0 = 222.0$

$v_f = 14.9\ m/s$

(e) Using your answers from part (c), verify your answer to part (d) by using a different method of calculation.

Use Theorem: Total Work = Change In Kinetic Energy

$+10{,}055 + 0 - 4144 = \frac{1}{2}(60.0)v_f^2 - \frac{1}{2}(60.0)(5.00)^2$

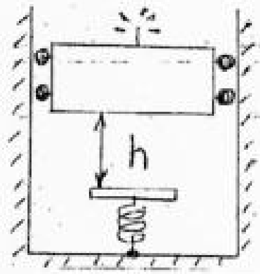
$5911 = 30v_f^2 - 750$

$v_f = \left\{\frac{5911 + 750}{30}\right\}^{1/2} = \underline{14.9\ m/s}$ ie part (d) Result Verified

④

Total 20

4. The cable of an 18000 N elevator snaps when the elevator is at rest with the bottom of the elevator a distance h = 4.00 metres above a cushioning spring whose spring constant is 150,000 N/m. When the cable snaps, a safety device on the side of the elevator is activated so that a constant frictional force of 3000 N opposes all subsequent motion of the elevator.



(a) Find the speed v of the elevator just before it hits the spring.

v = 8.08 m/s (6)

$$ME_{①} + W_{\text{by } f,\ ①\to②} = ME_{②} \quad \text{where } ME = K + U_{grav.}$$

$$(0 + mgh) + (-fd) = \left(\tfrac{1}{2}mv_2^2 + 0\right)$$

$$\frac{18000}{9.80}(9.80)(4.00) - (3000)(4.00) = \frac{1}{2}\left(\frac{18000}{9.80}\right)v_2^2$$

$$v_2^2 = \left(\frac{72{,}000 - 12{,}000}{18{,}000}\right)(2)(9.80) = 65.333$$

$$v_2 = 8.083$$

(b) Find the maximum distance d that the spring is compressed.

d = 1.00 m (7)

$$ME_{②} + W_{\text{by } f,\ ②\to③} = ME_{③} \quad \text{where } ME = K + U_g + U_s$$

OR $$ME_{①} + W_{\text{by } f,\ ①\to③} = ME_{③}$$

$$\{0 + mg(h+d) + 0\} + \{-f(h+d)\} = \{0 + 0 + \tfrac{1}{2}kd^2\}$$

$$18000(4+d) - 3000(4+d) = \tfrac{1}{2}(150{,}000)d^2$$

$$75{,}000d^2 - 15{,}000d - 60{,}000 = 0$$

$$5d^2 - d - 4 = 0 \Rightarrow d = \frac{1 \pm \sqrt{1+80}}{10} = +1.00 \text{ OR } -0.800$$

Choose +ve d for compression of spring (ie $h+d > h$)

(c) Find the distance L that the elevator will "bounce" back up the shaft as measured from the lowest point reached in part (b).

L = 3.57 m (7)

$$ME_{③} + W_{\text{by } f,\ ③\to④} = ME_{④}$$

where $ME = K + U_g + U_s$ but spring is not attached.

$$\{0 + 0 + \tfrac{1}{2}kd^2\} + (-fL) = \{0 + mgL + 0\}$$

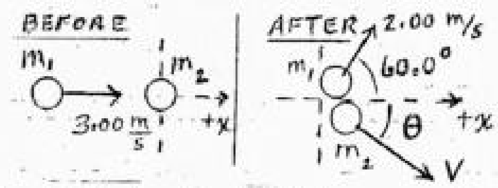
$$\tfrac{1}{2}(150{,}000)(1.00)^2 - 3000L = 18{,}000L$$

$$75{,}000 = 21{,}000L$$

$$L = \frac{75{,}000}{21{,}000} = 3.57 \text{ m}$$

Total 20

5. Mass $m_1 = 1.00$ kg, initially moving at a speed of 3.00 m/s collides with a second mass $m_2 = 2.00$ kg, initially at rest. After the collision m_1 is observed to move at 2.00 m/s in a direction of 60.0° from its original line of motion.



(a) Determine the magnitude and direction of the velocity of m_2 after the collision.

Conservation of Linear Momentum of System

V=	1.32	m/s
θ=	40.9	deg

10

(x) $m_1 v_{1i} + 0 = m_1 v_{1f} \cos 60^\circ + m_2 v_{2f} \cos\theta$

$(1.00)(3.00) = (1.00)(2.00)(0.500) + 2.00\, V\cos\theta$

$V\cos\theta = \dfrac{3.00 - 1.00}{2.00} = 1.00$ ①

(y) $0 + 0 = m_1 v_{1f} \sin 60^\circ - m_2 v_{2f} \sin\theta$

$0 = (1.00)(2.00)(0.86603) - 2.00\, V \sin\theta$

$V\sin\theta = \dfrac{1.73205}{2.00} = 0.86603$ ②

Eqn ② ÷ ① $\Rightarrow \tan\theta = 0.86603 \Rightarrow \theta = 40.89^\circ$

From ① $V = \dfrac{1.00}{\cos 40.89} = 1.323$ m/s

(b) Determine the change in kinetic energy of the system of the two masses during the collision.

ΔK = −0.75 J

5

$\Delta K = K_f - K_i$

$= \left(\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2\right) - \left(\frac{1}{2} m v_{1i}^2 - 0\right)$

$= \left\{\frac{1}{2}(1.00)(2.00)^2 + \frac{1}{2}(2.00)(1.323)^2\right\} - \left\{\frac{1}{2}(1.00)(3.00)^2\right\}$

$= \{2.00 + 1.75\} - \{4.50\}$

$= 3.75 - 4.50 = -0.75$ J

(c) Determine the magnitude and direction ϕ (relative to the positive x axis) of the average force exerted on m_1 by m_2 during the collision, if the duration of the collision is 0.0350s.

Alt-Method: Calculate $\dfrac{\Delta p_1}{\Delta t}$

F_{ave} =	75.6	N
φ=	139	deg

5

$\underset{\text{on } m_1}{F_{ave}} = -\underset{\text{on } m_2}{F_{ave}}$

$\underset{\text{on } m_2}{F_{ave}} = \dfrac{\Delta p_2}{\Delta t} = \dfrac{p_{2f} - p_{2i}}{\Delta t} = \dfrac{m_2 V - 0}{\Delta t}$

$\left|\underset{\text{on } m_2}{F_{ave}}\right| = \dfrac{(2.00)(1.323)}{0.0350}$ directed at angle $\theta = 40.9^\circ$ below the x axis.

$= 75.59$ N

Direction of $\underset{\text{on } m_1}{F_{ave}}$ is opposite to $\underset{\text{on } m_2}{F_{ave}}$, at an

angle $\phi = 180^\circ - 40.89^\circ = 139.1^\circ$ counter clockwise from the +ve x axis

Total 20

6. Professors Brandon and O'Donovan go into outer space together to experience zero gravity and to perform some experiments for a UW Physics demonstration video. Each professor with his space suit has mass 120.0 kg and they hold onto opposite ends of a 60.0 kg rod of length 8.00 m. Each professor is initially rotating about the centre of mass of the system at a speed of 6.00 m/s. $I_c(\text{rod}) = ML^2/12$

(a) What is their angular speed, ω?

$v = R\omega$

$\omega = \frac{v}{R} = \frac{6.00}{4.00} = 1.50 \text{ rad/s}$

ω = 1.50 rad/s

2

(b) If the two experimenters are treated as point masses, what is the initial total angular momentum L of the system?

$L_{system_i} = I_{system_i}\omega_i$ where $I_{system} = I_{c_{rod}} + 2mR^2$

L = 6240 kg m²/s

5

$L_{s_i} = \left[\frac{1}{12}ML^2 + 2m\left(\frac{L}{2}\right)^2\right]\omega$

$= \left[\frac{1}{12}(60.0)(8.00)^2 + 2(120)(4)^2\right](1.50)$

$= [320 + 3840](1.50) = (4160)(1.50) = 6240 \text{ kg m}^2/\text{s}$

(c) What is the initial kinetic energy K of the system?

$K_i = \frac{1}{2}I_i\omega_i^2$

$= \frac{1}{2}(4160)(1.50)^2$

$= 4680 \text{ J}$

K = 4680 J

3

Alt Method: $K = \frac{1}{2}I_{rod}\omega_i^2 + 2\left(\frac{1}{2}mv_i^2\right)$

(d) By pulling on the rod, they both move toward the centre at the same rate until they are separated by 5.00 m. What is their new speed, v?

Forces are internal to system, so apply conservation of angular momentum

v = 8.57 m/s

5

$L_i = L_f$

$6240 = I_f\omega_f = \left[\frac{1}{12}ML^2 + 2m\left(\frac{5.00}{2}\right)^2\right]\omega_f$

$= [320 + 1500]\omega_f = 1820\,\omega_f$

$\omega_f = 6240/1820 = 3.429 \text{ rad/s}$

$v' = R'\omega_f = (2.50)(3.429) = 8.571 \text{ m/s}$

(e) How much total work did the two experimenters perform in this operation?

Since the only forces that perform work are provided by the two experimenters, $W_{Total} = \Delta K = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$

W = 6020 J

5

$W_{Tot} = \frac{1}{2}[1820](3.429)^2 - \frac{1}{2}[4160](1.50)^2$

$= 10{,}697 - 4680 = 6017 \text{ J}$

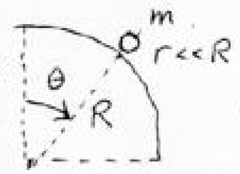
Total 20

7. A student places a small rubber ball on top of the Physics Building Observatory dome. The student gives it a gentle push to start it rolling. Assume that it rolls without slipping. Treat the dome as a perfect hemisphere of radius R = 2.00m. The ball is a solid sphere of mass m and radius r and with moment of inertia

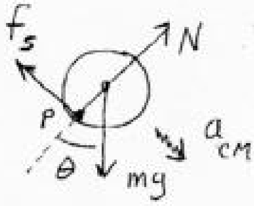
$$I_C = \frac{2}{5} mr^2.$$

(a) In the space provided, draw a free-body diagram of the ball when it has rolled down the dome through an angle θ measured from the vertical. Show all external forces acting on the ball.

Sketch of Problem:



FBD at Angle θ:



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(b) Find an expression for the tangential acceleration of the centre of mass of the rolling ball as a function of θ (i.e. find its acceleration component tangent to the surface of the dome).

Find torques about instantaneous rotation axis at P

$\sum \tau_P = I_P \alpha_P$ (+)

$r mg \sin\theta = I_P \alpha_P = \left[\frac{2}{5}mr^2 + mr^2\right]\alpha_P$

But $a_{cM} = r\alpha_P$

$r mg \sin\theta = \frac{7}{2} mr^2 \frac{a_c}{r} \Rightarrow g\sin\theta = \frac{7}{5}a_c \Rightarrow \underline{a_c = \frac{5}{7}g\sin\theta}$ 5

(c) Use energy methods to find an expression for the speed of the centre of mass of the ball as a function of angle θ. (Neglect the small radius r of the ball in comparison with the large radius R of the dome).

Since f_s force does no work and all other forces are conservative,

Use $ME_{Top} = ME_{at\,\theta}$ where $ME = U_{grav} + K_{rolling}$

$mgR + 0 = mgR\cos\theta + \left[\frac{1}{2}mv_c^2 + \frac{1}{2}I_c\omega^2\right]$

Using $v_c = r\omega$, $mgR(1-\cos\theta) = \frac{1}{2}mv_c^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v_c^2}{r^2}$

$= \left(\frac{1}{2}+\frac{1}{5}\right)mv_c^2 = \frac{7}{10}mv_c^2$

$v_c = \left[\frac{10}{7}gR(1-\cos\theta)\right]^{1/2}$

$= \left[\frac{10}{7}(9.80)(2.00)(1-\cos\theta)\right]^{1/2} = 5.29\sqrt{1-\cos\theta}$ 6

(d) Find the special angle θ′ at which the ball leaves the surface of the dome?

$\boxed{\theta' = 54.0 \text{ deg}}$

From FBD, forces in radial direction give

$mg\cos\theta - N = m\frac{v_c^2}{R}$ (A) Neglect r in (R+r) denominator since r<<R 6

While rolling $v_c^2 = \frac{10}{7}gR(1-\cos\theta)$ which increases as θ increases, and in eqn (A) we reach angle θ′ where N→0.

i.e. $mg\cos\theta' - 0 = m\frac{10}{7}gR(1-\cos\theta')/R$

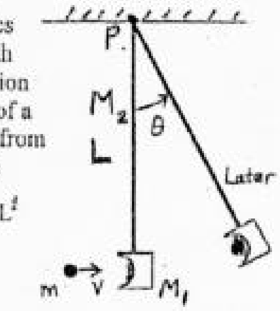
$\cos\theta' = \frac{10}{7}(1-\cos\theta')$

$17\cos\theta' = 10 \Rightarrow \cos\theta' = \frac{10}{17} = 0.58824$

$\theta' = 53.97°$

Total 20

8.(a) Professor Brandon's grandson, Matthew, is testing a spring gun for a physics demonstration. On release, a steel ball (mass m = 10.0g) leaves the gun with horizontal speed v = 12.0 m/s. The ball makes a completely inelastic collision with some plasticene in a cup (total mass M_1 = 140 g), attached to one end of a uniform rod (mass M_2 = 300 g, length L = 40.0 cm) which hangs vertically from a frictionless pivot, P. After the collision, the rod swings in a vertical plane through an angle θ_{Max} before momentarily coming to rest. I_c (rod) = $\frac{1}{12}ML^2$



(i) What is the total moment of inertia about P of the rod, cup, plasticene and steel ball? (Treat the cup/plasticene/ball combination as a point mass.)

$$I_p = \sum m_i r_i^2 = I_{rod} + (I_{cup} + I_{pl} + I_{ball})$$
$$= \left[\tfrac{1}{12}M_2L^2 + M_2\left(\tfrac{L}{2}\right)^2\right] + (M_1 + m)L^2$$
$$= \tfrac{1}{3}(0.3)(0.4)^2 + (0.14)(0.4)^2 + (0.01)(0.4)^2$$
$$= 0.0160 + 0.0224 + 0.0016 = 0.0400\ kg\ m^2$$

$\boxed{I_p = 0.0400\ \text{kg m}^2}$ OR $4.00 \times 10^{-2}\ kg\,m^2$ (3)

(ii) Find the angular speed of the system immediately after the collision.

Since forces are internal during the collision, use

$L_{i_P} = L_{f_P}$ $L_i = r \times m v_i$ for the ball.

$$L m v + 0 = I_p \omega$$
$$(0.4)(0.010)(12.0) = (0.0400)\,\omega$$
$$\omega = 0.0480/0.0400 = 1.20\ rad/s$$

$\boxed{\omega = 1.20\ \text{rad/s}}$ (3)

(iii) What is the value of θ_{Max}?

$ME_{Bot} = ME_{Top}$ where $ME = K_{rot} + U_{grav}$.

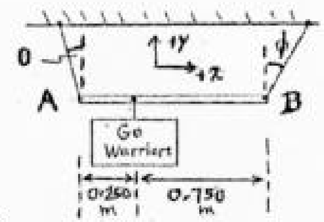
$$\tfrac{1}{2}I_p\omega^2 + 0 + 0 = 0 + M_2 g \tfrac{L}{2}(1-\cos\theta_M) + (M_1+m)gL(1-\cos\theta_M)$$

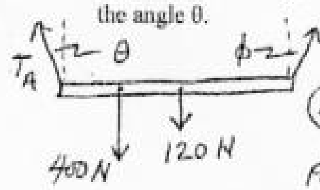
where we have used CM_{rod} rises $\Delta y = \frac{L}{2}(1-\cos\theta)$, cup rises $L(1-\cos\theta)$

$$0.02880 = (1-\cos\theta_M)\left[0.3(9.80)(0.4/2) + (0.14+0.01)(9.80)(0.4)\right]$$
$$1-\cos\theta_M = 0.0288/[0.588 + 0.588] = 0.02449$$
$$\cos\theta_M = 0.9755 \Rightarrow \theta = 12.7°$$

$\boxed{\theta_M = 12.7\,°}$ (4)

(b) At the Naismith Classic, a Waterloo fan hangs a banner of weight 400 N from a uniform rod AB which is 1.00 m long and of weight 120 N. The rod is suspended from the gym ceiling in a horizontal position by two ropes as shown, and it is in equilibrium with the rope at B making an angle of φ = 36.9°. Find the tensions T_A and T_B in the two ropes, and determine the angle θ.





$\sum F_x$ (1) $T_B \sin\phi - T_A \sin\theta = 0$

$\sum F_y$ (2) $T_B \cos\phi + T_A \cos\theta = 520$

For torque eqn, choose origin at A to eliminate T_A torque. $\sum \tau_{zA}$ (3)

$$(1.00)T_B\cos\phi - (0.25)(400) - (0.50)(120) = 0$$

(3) $\Rightarrow T_B = (100 + 60)/\cos 36.9° = 200.08$

(1) $\Rightarrow T_A \sin\theta = 200 \sin 36.9° = 120.1$

(2) $\Rightarrow T_A \cos\theta = 520 - 200\cos 36.9° = 360.0$

$$\tan\theta = 120/360 = \tfrac{1}{3} \Rightarrow \theta = 18.45°$$

and $T_A = 120/\sin 18.45° = 379.1\ N$

T_A = 379	N
T_B = 200	N
θ = 18.5	deg

(10)

Total 20