MATH 136 **Assignment 5**
Fall/05

Due: Thursday Oct. 27/05
(Show details of your work. Grade is out of 40.)

1 page 116 #4

4 marks

$$A - 5I_3 = \begin{pmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -1 & 3 \\ -8 & 2 & -6 \\ -4 & 1 & 3 \end{pmatrix}$$

$$(5I_3)A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} A = \begin{pmatrix} 45 & -5 & 15 \\ -40 & 35 & -30 \\ -20 & 5 & 40 \end{pmatrix}$$

2 page 116 #6

4 marks

(a) by linear combinations of the columns:

$$Ab_1 = \begin{pmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{pmatrix} * \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -2 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 13 \end{pmatrix}$$

$$A\mathbf{b_2} = \begin{pmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{pmatrix} * \begin{pmatrix} 3 \\ -1 \end{pmatrix} = 3\begin{pmatrix} 4 \\ -3 \\ 3 \end{pmatrix} - 1\begin{pmatrix} -2 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 14 \\ -9 \\ 4 \end{pmatrix}$$

so

$$AB = \begin{pmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{pmatrix}$$

(b)by row-column rule:

$$AB = \begin{pmatrix} 4*1-2*2 & 4*3-2*(-1) \\ -3*1+0*2 & -3*3 \\ 3*1+5*2 & 3*3+5*(-1) \end{pmatrix} = \begin{pmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{pmatrix}$$

3 page 117 #18

4 marks

Suppose the columns of B are denoted using MATLAB notation:

$$B = (\, B_{:1} \quad B_{:2} \quad B_{:3} \quad \ldots \quad B_{:n}\,).$$

If the product AB is defined, we claim that the first two columns of AB are equal. This holds because the j-th column of AB is a linear combination of the columns of A using the coefficients of the j-th column of B, i.e. $(AB)_{:j} = \sum_i B_{ij} A_{:,i}$. Therefore, we have $(AB)_{:1} = \sum_i B_{i1} A_{:,i} = (AB)_{:2} = \sum_i B_{i2} A_{:,i}$ since the two columns are equal, i.e. $B_{i1} = B_{i2}, \forall i$.

4 page 126 #8

6 marks

If $AD = I$, sincce A invertible, we multiply both sides of the equation from the left by A^{-1},so the equality becomes:

$$\begin{aligned} A^{-1}AD &= A^{-1}I \\ ID &= A^{-1}, \quad \text{by definition of the inverse and } I \\ D &= A^{-1}, \quad \text{by definition of } I \end{aligned}$$

5 page 126 #16

6 marks

$$\begin{aligned} AB &= C, \quad \text{by definition} \\ ABB^{-1} &= CB^{-1} \quad \text{since } B \text{ is invertible} \\ A &= CB^{-1}, \quad \text{by definition of the inverse and } I \\ A^{-1} &= BC^{-1}, \quad \text{from Theorem 6,} \end{aligned}$$

i.e. the inverse is well defined and so exists.

6 page 127 #32

(and use a check column) ***5 marks***

A MATLAB program follows:

```
!rm output
diary output
clear all
A=[1 -2 1 1 0 0;
   4 -7 3 0 1 0;
  -2 6 -4 0 0 1]
A(2,:)=A(2,:)-A(2,1)*A(1,:)
A(3,:)=A(3,:)-A(3,1)*A(1,:)
```

```
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(1,:)=A(1,:)-A(1,2)*A(2,:)
diary off
```

and the output is:

```
A =

     1    -2     1     1     0     0
     4    -7     3     0     1     0
    -2     6    -4     0     0     1


A =

     1    -2     1     1     0     0
     0     1    -1    -4     1     0
    -2     6    -4     0     0     1


A =

     1    -2     1     1     0     0
     0     1    -1    -4     1     0
     0     2    -2     2     0     1


A =

     1    -2     1     1     0     0
     0     1    -1    -4     1     0
     0     0     0    10    -2     1


A =

     1     0    -1    -7     2     0
     0     1    -1    -4     1     0
     0     0     0    10    -2     1
```

We cannot row reduce to I. A does NOT have an inverse.

7 page 127 #33

5 marks

A MATLAB file follows:

```
A=[
1 0 0 1 0 0 ;
1 1 0 0 1 0 ;
1 1 1 0 0 1]
A(1,:)=A(1,:)/A(1,1)
A(2,:)=A(2,:)-A(2,1)*A(1,:)
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(3,:)=A(3,:)/A(3,3)
```

With output:

```
A =

     1     0     0     1     0     0
     1     1     0     0     1     0
     1     1     1     0     0     1


A =

     1     0     0     1     0     0
     1     1     0     0     1     0
     1     1     1     0     0     1


A =

     1     0     0     1     0     0
     0     1     0    -1     1     0
     1     1     1     0     0     1


A =

     1     0     0     1     0     0
     0     1     0    -1     1     0
     0     1     1    -1     0     1


A =

     1     0     0     1     0     0
     0     1     0    -1     1     0
     0     1     1    -1     0     1
```

```
A =

     1     0     0     1     0     0
     0     1     0    -1     1     0
     0     0     1     0    -1     1


A =

     1     0     0     1     0     0
     0     1     0    -1     1     0
     0     0     1     0    -1     1
```

so the inverse of $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ is $\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ for the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

Next, the inverse is computed with following matlab program:

```
A=[
1 0 0 0 1 0 0 0
1 1 0 0 0 1 0 0
1 1 1 0 0 0 1 0
1 1 1 1 0 0 0 1]
A(1,:)=A(1,:)/A(1,1)
A(2,:)=A(2,:)-A(2,1)*A(1,:)
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(4,:)=A(4,:)-A(4,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(4,:)=A(4,:)-A(4,2)*A(2,:)
A(3,:)=A(3,:)/A(3,3)
A(4,:)=A(4,:)-A(4,3)*A(3,:)
A(4,:)=A(4,:)/A(4,4)


A =

     1     0     0     0     1     0     0     0
     1     1     0     0     0     1     0     0
     1     1     1     0     0     0     1     0
     1     1     1     1     0     0     0     1


A =
```

```
     1     0     0     0     1     0     0     0
     1     1     0     0     0     1     0     0
     1     1     1     0     0     0     1     0
     1     1     1     1     0     0     0     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     1     1     1     0     0     0     1     0
     1     1     1     1     0     0     0     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     1     1     0    -1     0     1     0
     1     1     1     1     0     0     0     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     1     1     0    -1     0     1     0
     0     1     1     1    -1     0     0     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     1     1     0    -1     0     1     0
     0     1     1     1    -1     0     0     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     0     1     0     0    -1     1     0
     0     1     1     1    -1     0     0     1
```

```
A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     0     1     0     0    -1     1     0
     0     0     1     1     0    -1     0     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     0     1     0     0    -1     1     0
     0     0     1     1     0    -1     0     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     0     1     0     0    -1     1     0
     0     0     0     1     0     0    -1     1


A =

     1     0     0     0     1     0     0     0
     0     1     0     0    -1     1     0     0
     0     0     1     0     0    -1     1     0
     0     0     0     1     0     0    -1     1
```

so the inverse of $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ is $\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$;

Now we guess the form of the inverse B of A, is: $B_{ii} = 1, B_{i+1,i} = -1, i = 1, 2, \ldots, n-1$, with zeros elsewhere. We claim that $AB = BA = I$. Use the row-column rule. We see that only the nonzero diagonal elements overlap in the i-th row of A and i-th column of B, i.e. $(AB)_{ii} = 1$. And, checking the overlaps, we see $(AB)_(ij) = -1 * 1 + 1 * 1 = 0, i \neq j$, i.e. $AB = I$. Similarly, we can check $BA = I$.

8 page 132 #6, #8

6 marks

(#6)
```
A =

     1    -5    -4     1     0     0
     0     3     4     0     1     0
    -3     6     0     0     0     1

A =

     1    -5    -4     1     0     0
     0     3     4     0     1     0
     0    -9   -12     3     0     1
```

After the single pivot operation to obtain 0 in the $3,1$ entry, we see that the next pivot to obtain 0 in the $3,2$ entry results in a zero row in the original A, so there are not enough pivot elements and the matrix is NOT invertible.

(#8) The matrix is invertible by Theorem 8, since there are $n = 4$ pivot position in this already triangular matrix. so it is invertible