MATH 136 Solutions Assignment 2 Fall/05

September 30, 2005

Following are the solutions. Most of the solutions are given using MATLAB. However, you are not required to use MATLAB for these assignments.

Contents

Due: Thursday Sept. 29/05
(Show details of your work. Grade is out of 30.)

1 page 25 #14 (use a check column)

5 marks

Solution. the matlab commands follow (a check colum is included):

```
echo on
A=[
1 2 -5 -6 0 -5 -13
0 1 -6 -3 0 2 -6
0 0 0 0 1 0 1
0 0 0 0 0 0 0]
A(1,:)=A(1,:)-A(1,2)*A(2,:)
echo off
```

and the output is:

```
A =

     1     2    -5    -6     0    -5   -13
     0     1    -6    -3     0     2    -6
     0     0     0     0     1     0     1
     0     0     0     0     0     0     0


A =

     1     0     7     0     0    -9    -1
     0     1    -6    -3     0     2    -6
     0     0     0     0     1     0     1
     0     0     0     0     0     0     0
```

then we can read off the solution using the second to last column (i.e. ignore the check column) and the columns corresponding to the free variables:

$$\begin{aligned} x_1 &= -9 - 7x_3 \\ x_2 &= 2 + 6x_3 + 3x_4 \\ x_5 &= 0 \end{aligned}$$

and x_3,x_4 are free.

2 page 26 #24

4 marks

Solution. No. If the fifth colum is a valid pivot column, then it means it is the leading entry of this row. So there is a row like:

$$(0 \quad 0 \quad 0 \quad 0 \quad b),$$

where b is a non-zero entry. Thus it has no solution; the system is inconsistent. (See Theorem 2, page 24.)

3 page 27 #34

5 marks

Solution. the matlab program is as follows:

```
clear all
v=[0 2 4 6 8 10]'
vo=v;
A=[
 v.^5  v.^4 v.^3 v.^2 v.^1 ones(6,1)
]
f=[0 2.90 14.8 39.6 74.3 119]'
disp('the coefficients using the linear system Ac=f are:')
coeff=(A\f);
coeff'
disp('the coefficients using the matlab command polyfit are:')
p=polyfit(v,f,5)
v=7.5;
disp('the force at 7.50 x 100 ft/sec using inner-product is:')
 [v.^5  v.^4 v.^3 v.^2 v.^1 1]*coeff
disp('the force at 7.50 x 100 ft/sec using polyval is:')
x=polyval(p,7.5)
%%%%%%%%%%try cubic polynomial
disp('coefficient matrix for cubic polynomial is')
v=vo;
A=[
  v.^3 v.^2 v.^1 ones(6,1)
]
disp('From the RREF of [A f] we see that the system is inconsistent.')
disp('There is no cubic polynomial that fits the data.')
rref([A f])
```

the output follows:

```
v =

     0
     2
     4
     6
     8
    10


A =
```

```
           0           0           0           0           0           1
          32          16           8           4           2           1
        1024         256          64          16           4           1
        7776        1296         216          36           6           1
       32768        4096         512          64           8           1
      100000       10000        1000         100          10           1


f =

         0
    2.9000
   14.8000
   39.6000
   74.3000
  119.0000

the coefficients using the linear system Ac=f are:

ans =

    0.0026   -0.0701    0.6615   -1.1948    1.7125         0

the coefficients using the matlab command polyfit are:

p =

    0.0026   -0.0701    0.6615   -1.1948    1.7125   -0.0000

the force at 7.50 x 100 ft/sec using inner-product is:

ans =

   64.8384

the force at 7.50 x 100 ft/sec using polyval is:

x =

   64.8384

coefficient matrix for cubic polynomial is

A =

           0           0           0           1
```

```
           8              4              2              1
          64             16              4              1
         216             36              6              1
         512             64              8              1
        1000            100             10              1

From the RREF of [A f] we see that the system is inconsistent.
There is no cubic polynomial that fits the data.

ans =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
     0     0     0     0     0
```

4 page 37 #10

2 marks

Solution. the vector equation is:

$$x_1 \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ -7 \\ 6 \end{pmatrix} + x_3 \begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 15 \end{pmatrix}$$

5 page 38 #14

4 marks

Solution. Note: applying row reduction to the augmented matrix shows whether the system is consistent; equivalently, this shows whether b is a linear combination of columns of A. the matlab program to solve $Ax = b$ is:

```
echo on
 A=[1 -2 -6 11
0 3 7 -5
1 -2 5 9]
A(3,:)=A(3,:)-A(3,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(3,:)=A(3,:)/A(3,3)
A(1,:)=A(1,:)-A(1,3)*A(3,:)
A(2,:)=A(2,:)-A(2,3)*A(3,:)
A(1,:)=A(1,:)-A(1,2)*A(2,:)
```

```
echo off
```

,

the results are as follows:

```
A =

     1    -2    -6    11
     0     3     7    -5
     1    -2     5     9


A =

     1    -2    -6    11
     0     3     7    -5
     0     0    11    -2


A =

    1.0000   -2.0000   -6.0000   11.0000
         0    1.0000    2.3333   -1.6667
         0         0   11.0000   -2.0000


A =

    1.0000   -2.0000   -6.0000   11.0000
         0    1.0000    2.3333   -1.6667
         0         0   11.0000   -2.0000


A =

    1.0000   -2.0000   -6.0000   11.0000
         0    1.0000    2.3333   -1.6667
         0         0    1.0000   -0.1818


A =

    1.0000   -2.0000         0    9.9091
         0    1.0000    2.3333   -1.6667
         0         0    1.0000   -0.1818
```

```
A =

    1.0000   -2.0000         0    9.9091
         0    1.0000         0   -1.2424
         0         0    1.0000   -0.1818


A =

    1.0000         0         0    7.4242
         0    1.0000         0   -1.2424
         0         0    1.0000   -0.1818
```

The last column of the augmented matrix is NOT a pivot column. Therefore the system is consistent, i.e. b IS a linear combination of the columns. Equivalently, b is in the span of the columns of A.

6 page 47 #4

2 marks

Solution. By the definition of the first product rule: $Ax = 1\begin{pmatrix}8\\5\end{pmatrix} + 1\begin{pmatrix}3\\1\end{pmatrix} + 1\begin{pmatrix}-4\\2\end{pmatrix} = \begin{pmatrix}7\\8\end{pmatrix}$; by the row-vector rule, the product will be: $\begin{pmatrix}8*1+3*1+(-4)*1\\5*1+1*1+2*1\end{pmatrix} = \begin{pmatrix}7\\8\end{pmatrix}$. Both ways yield the same result.

7 page 47 #12

4 marks

Solution. The Matlab program follows:

```
A=[
1 2 1 0
-3 -1 2 1
0 5 3 -1]
A(2,:)=A(2,:)-A(2,1)*A(1,:)
A(2,:)=A(2,:)/A(2,2)
A(3,:)=A(3,:)-A(3,2)*A(2,:)
A(3,:)=A(3,:)/A(3,3)
A(1,:)=A(1,:)-A(1,3)*A(3,:)
A(2,:)=A(2,:)-A(2,3)*A(3,:)
A(1,:)=A(1,:)-A(1,2)*A(2,:)
disp('the solution is')
x=A(:,4)
```

the results are as follows:

```
A =

     1     2     1     0
    -3    -1     2     1
     0     5     3    -1


A =

     1     2     1     0
     0     5     5     1
     0     5     3    -1


A =

    1.0000    2.0000    1.0000         0
         0    1.0000    1.0000    0.2000
         0    5.0000    3.0000   -1.0000


A =

    1.0000    2.0000    1.0000         0
         0    1.0000    1.0000    0.2000
         0         0   -2.0000   -2.0000


A =

    1.0000    2.0000    1.0000         0
         0    1.0000    1.0000    0.2000
         0         0    1.0000    1.0000


A =

    1.0000    2.0000         0   -1.0000
         0    1.0000    1.0000    0.2000
         0         0    1.0000    1.0000


A =
```

```
    1.0000    2.0000         0   -1.0000
         0    1.0000         0   -0.8000
         0         0    1.0000    1.0000


A =

    1.0000         0         0    0.6000
         0    1.0000         0   -0.8000
         0         0    1.0000    1.0000

the solution is

x =

    0.6000
   -0.8000
    1.0000
```

8 page 48 #18

4 marks

Solution. The columns of B span $\Re^4$ if and only if B has a pivot position in each row. so we need only observe the pivots during row reduction. The MATLAB program is as follows:

```
B=[
1 3 -2 2
0 1 1 -5
1 2 -3 7
-2 -8 2 -1]
B(3,:)=B(3,:)-B(3,1)*B(1,:)
B(4,:)=B(4,:)-B(4,1)*B(1,:)
B(2,:)=B(2,:)/B(2,2)
B(3,:)=B(3,:)-B(3,2)*B(2,:)
B(4,:)=B(4,:)-B(4,2)*B(2,:)
B(3,:)=B(3,:)/B(3,3)
B(4,:)=B(4,:)-B(4,3)*B(3,:)
B(4,:)=B(4,:)/B(4,4)
```

the results are as following:

```
B =

     1     3    -2     2
     0     1     1    -5
```

```
     1     2    -3     7
    -2    -8     2    -1


B =

     1     3    -2     2
     0     1     1    -5
     0    -1    -1     5
    -2    -8     2    -1


B =

     1     3    -2     2
     0     1     1    -5
     0    -1    -1     5
     0    -2    -2     3


B =

     1     3    -2     2
     0     1     1    -5
     0    -1    -1     5
     0    -2    -2     3


B =

     1     3    -2     2
     0     1     1    -5
     0     0     0     0
     0    -2    -2     3



B =

     1     3    -2     2
     0     1     1    -5
     0     0     0     0
     0     0     0    -7

Warning: Divide by zero.
(Type "warning off MATLAB:divideByZero" to suppress this warning.)
```

The third row has no pivot. So the columns of B cannot span $\Re^4$. Thus, $Ax = b$ does not have a solution for each $b \in \Re^4$.

Polynomial Approximation

A MATLAB Program for Polynomial Interpolation

Following is a program that generates random data and finds the polynomial to fit the data. A plot is generated.

```
clear all
clf
n=10;              % number of points
m=n-1;             % degree of polynomial
disp(['number of points and degree of polynomial:  ',num2str([n m])])
v=sort((10*randn(n,1)))
rhs=((10*randn(n,1)))
%keyboard
disp('coefficient matrix A is found to be')
A=ones(n,1)
for i=1:m,
A=[v.^i A]
end
A
disp('solve for polynomial coefficients')
coeff=A\rhs
disp(['evaluate the polynomial at the point:  ',num2str(mean(v))])
polyval(coeff,mean(v))
disp('plot the polynomial and mark the data points with x')
x=v(1):.1:v(end);
y=polyval(coeff,x);
plot(x,y)
hold on
plot(v,rhs,'kx')
hold off
```