Math 138 Physics Based Section Assignment 2

(Q1) Find the Laplace transform, $F(s)$ of each of the following functions ($a > 0$ is a parameter):
i) $f(t) = t^2 + 2t$
ii) $f(t) = t\exp(-at)$ using the rule for multiplying by the exponential from the notes
ii) $f(t) = t\exp(-at)$ using the transform of $\exp(-at)$ and differentiating with respect to a
iii) $f(t) = t\sin(at)$
iv) $f''(t)$ for a general function $f(t)$ (call the transfrom of $f(t)$, $\mathcal{L}(f(t)) = F(s)$).
v) $H(t-1)$ where $H(.)$ is the Heaviside step function.
vi) $tH(t-1)$ where $H(.)$ is the Heaviside step function.
vii) $t(1 - H(t-1))$ where $H(.)$ is the Heaviside step function.
viii) $\cos^2(3t)$

(Q2) Find the inverse Laplace transform using the method of partial fractions or by converting to a known form (use Q1 and your notes).
i)

$$F(s) = \frac{3s+2}{s^2+5s+6}$$

ii)

$$F(s) = \frac{3s+2}{s^3+5s^2+6s}$$

iii)

$$F(s) = \exp(-s)/s$$

iv) (BONUS)

$$F(s) = \exp(-s)/(s+3)$$

v)

$$F(s) = \frac{s+4}{(s+4)^2+3^2}$$

vi)

$$F(s) = \frac{3}{s^2+8s+25}$$

(Q3) Solve the following initial value problems for $y(t)$ using the Laplace transform.
i) $y' + y = 4$, $y(0) = 3$
ii) $y' + y = 4$, $y(0) = -3$
iii) $y' + y = t$, $y(0) = 3$
iv) $y' + y = \exp(-t)$, $y(0) = 3$

v) $y' + y = \exp(-2t)$, $y(0) = 3$

(Q4) Consider the mass, M, of a spherical planet of radius R, problem. The density as a function of radius is given by $d(r)$.
i) Find the mass of a planet with $d(r) = (1 + 0.05 \sin(\frac{r}{R}))$. Use Maple for the messy integral you find.
ii) Find the mass of a planet for which $d(r) = 1 - 0.25H(r - 0.5R)$.
iii) For a general $d(r)$ the mass can be written as a function of the planet's radius as $M(R)$. Find $M'(R)$ and explain what it means.

(Q5) Consider a mass m attached to a linear spring with spring constant k. The mass is placed on a frictionless table. i) Use Newton's law to derive the simple harmonic oscillator equation, or SHO, that governs the motion of the mass. Let $x(t)$ be the position of the mass and let the origin of your coordinates be placed at the position the mass occupies when the spring is not stretched.
ii) Solve the initial value problem $x(0) = 0.2$, $x'(0) = 0$.
iii) Solve the initial value problem $x(0) = 0.0$, $x'(0) = 0.2$. Comment on the difference between the two problems.
iv) Multiply the equation of motion by $x'(t)$ and integrate with respect to t. You will need to use the Chain Rule.
v) In the expression you found in iv identify the potential and kinetic energy terms and thus argue that you have derived the conservation of energy.
vi) Now say the mass experiences some external forcing, which we will label $F_e(t)$. Show that the same governing equation still applies but the right hand side is now non-zero.
vii) Rederive the conservation of energy equation and explain how the total energy changes.

(Q6) Consider the curve given by $y = x^{3/2}$ on the interval $[1, 2]$.
i) Find the arclength using the formula from your course materials (page 17 and thereabout).
ii) Sketch the curve. Now divide the interval into 5 sub-intervals and estimate the arclength by using line segments to approximate the curve. How good is the approximation?