

Stone in Well

a) A boy with a stopwatch drops a stone in a well. If he hears the splash a time t after dropping the stone how deep, d , is the well?

Let time taken by stone to reach bottom = x .

$$\text{Then } \Rightarrow d = \frac{1}{2}g\left(t - \frac{d}{v_w}\right)^2$$

$$\Rightarrow d = \frac{1}{2}gt^2 - \frac{d}{v_w}gt + \frac{gd^2}{2v_w^2}$$

$$\Rightarrow 0 = \frac{1}{2d}gt^2 - \frac{tg}{v_w} + \frac{gd}{2v_w^2}$$

Solving for d using matlab we get:

$$d = \frac{\pm(\sqrt{8gt^2v_w^4 + 4gt^2v_w^2 - 4g^2d^2v_w + gd^2} - 2gtv_w + gd)}{4v_w^2}$$

b) Consider the $v_w \gg gt$ limit of your solution for part (a) for both possible sign choices in front of the square root. What sign choice gives a reasonable answer and what is that answer? (Hint: Use the binomial expansion for the square root up to the second order term.)

$$\lim_{v_w \gg gt} d = \frac{1}{2}gt^2$$

d) What is the $g \rightarrow \infty$ limit of your answer to part (a)?

$$\lim_{g \rightarrow \infty} d = 0$$

e) If the boy assumed that sound travelled instantaneously what would he calculate the depth of the well to be?

$$d_{\text{inst.}} = \frac{1}{2}gt^2$$