SCAIS - electric circuit modelling

Keywords: ATP, Block diagram, Circuit modelling, Convex, PSIM, RLC Circuit, SCAIS, VMetodo.

This document presents the work done in an effort to implement electric circuit/network modelling with SCAIS.
The circuit chosen for demonstration is the one presented in the Electric ISA paper [1], displayed below.
The objective is to be able to model an electric circuit of low complexity (alternate current; linear elements) under the ISA[1] scope.
After implementing a valid circuit model it is desired the inclusion three protective breakers, A, B and C as displayed in fig.1, so that a security analysis can be performed.
The underlying concept is the switching of breakers that allows isolation of fault events. Displayed as example in fig.1 is the i8 switch that simulates a zero-impedance short circuit.

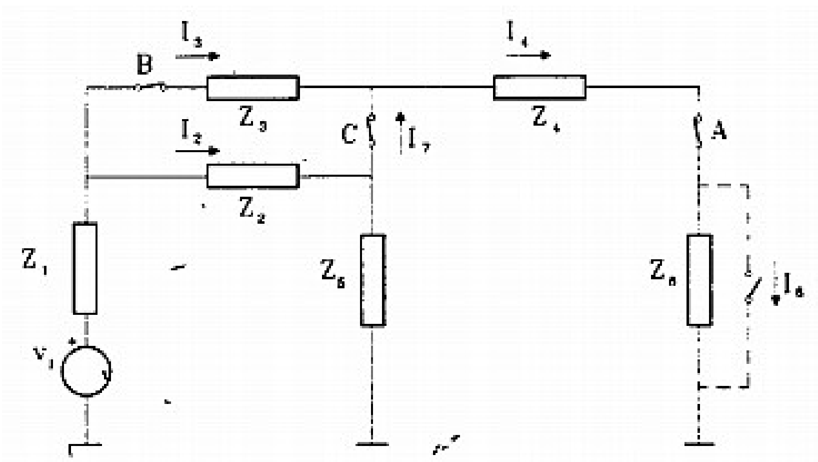

Fig.1 – Network selected for modelling.

Circuit parameters:

$v_1 = 1pu$ $\quad$ $Z_1 = 9.23x10^{-7}s$ $\quad$ $Z_4 = 1.56x10^{-2} + 2.371x10^{-5}s$ $\quad$ $Z_6 = 0.5 + 9.23x10^{-3}s$

$f = 50$ Hz $\quad$ $Z_2 = Z_3 = 3.6x10^{-5} + 4.138x10^{-7}s$ $\quad$ $Z_5 = \dfrac{0.1+9x10^{-3}s}{1+5x10^{-7}s+4.5x10^{-8}s^2}$

As the first step some first order[2] and simple RLC series circuits were modelled in order to sketch possible electric circuit/network models.
Some of the studied examples are presented.

Examples of study

Example 1

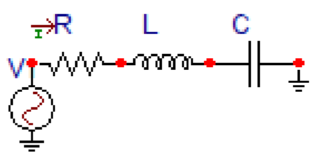

Fig2 – Example 1 network.

$v = 120V$ $\quad$ $f = 50Hz$
$R = 1000\Omega$ $\quad$ $L = 100mH$
$C = 5\mu F$

It is intended the determination of the current I.

1 Integrated Safety Assessment.
2 Circuits whose currents and voltages are described by first order differential equations. [2]

Example 2

This network was “extracted” from a branch of the case-study (fig.1) with the objective of determining I.

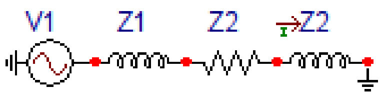

Fig3 – Example 2 network.

$v_1 = 1V$ $\qquad$ $f = 50Hz$

$Z_1 = sL_1 = 9.23 \times 10^{-7}s$ $\qquad$ $(L_1 = 9.23 \times 10^{-7}H)$

$Z_2 = R_2 + sL_2 = 3.6 \times 10^{-5} + 4.138 \times 10^{-7}s$

$(R_2 = 3.6 \times 10^{-5}\Omega;\ L_2 = 4.138 \times 10^{-7}H)$

Procedure - example1

There are some electric laws to take into account:

- Voltage and current across inductors (L) and capacitors (C):

$$i_c = C \times \frac{dv_c}{dt} \qquad i_L = \frac{1}{L}\int v_L\,dt \qquad v_L = L \times \frac{di_L}{dt} \qquad v_C = \frac{1}{C}\int i_C\,dt$$ (both v and i are time-dependant)

- Ohm's law:

$$R = \frac{v}{i} \quad \text{or} \quad Z = \frac{V}{I}$$

- Kirchhoff's voltage law:

$$\sum_{k=1}^{n} V_k = V_1 + V_2 + V_3 \ldots + V_n = 0$$

(The sum of the voltages across any closed loop equals zero)

Applying this to example 1 network yields: [where $v(t) = 1 \cdot \sin(w \cdot t)$, $w = 2 \cdot \pi \cdot f$]

$$v = v_R + v_L + v_C \Leftrightarrow v = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \Leftrightarrow \frac{1}{L}\frac{dv}{dt} = \frac{R}{L}\frac{di}{dt} + \frac{d^2 i}{dt^2} + \frac{1}{LC}i \Leftrightarrow \frac{d^2 i}{dt^2} = \frac{1}{L}\frac{dv}{dt} - \frac{R}{L}\frac{di}{dt} - \frac{1}{LC}i$$

To solve this differential equation it was used the Vmetodo module, performing a change of variable:

$$y_1' = i''$$
$$y_2 = i$$

This module is able to solve Ordinary Differential Equations[3] with maximum order of 4.

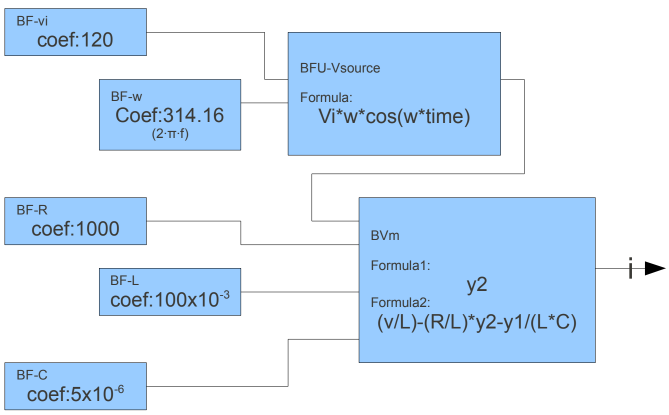

Fig4 – Xml implementation of example 1.

3 ODE: is a relation that contains functions of only one independent variable, and one or more of their derivatives with respect to that variable.[3,4]

This circuit was simulated with Babieca and the results obtained for i were validated both with PSIM and ATP software.

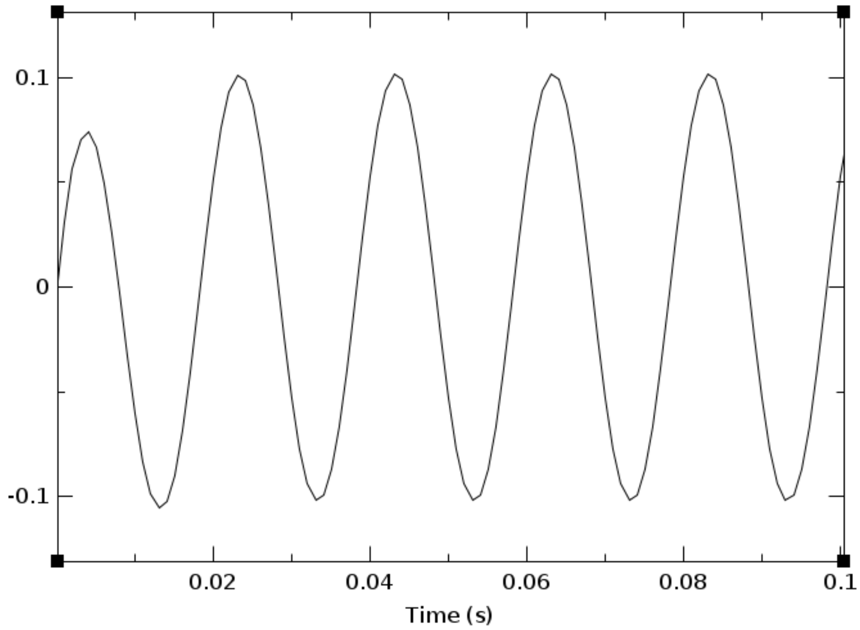

Fig5 – Current I of example 1 simulated with SCAIS.

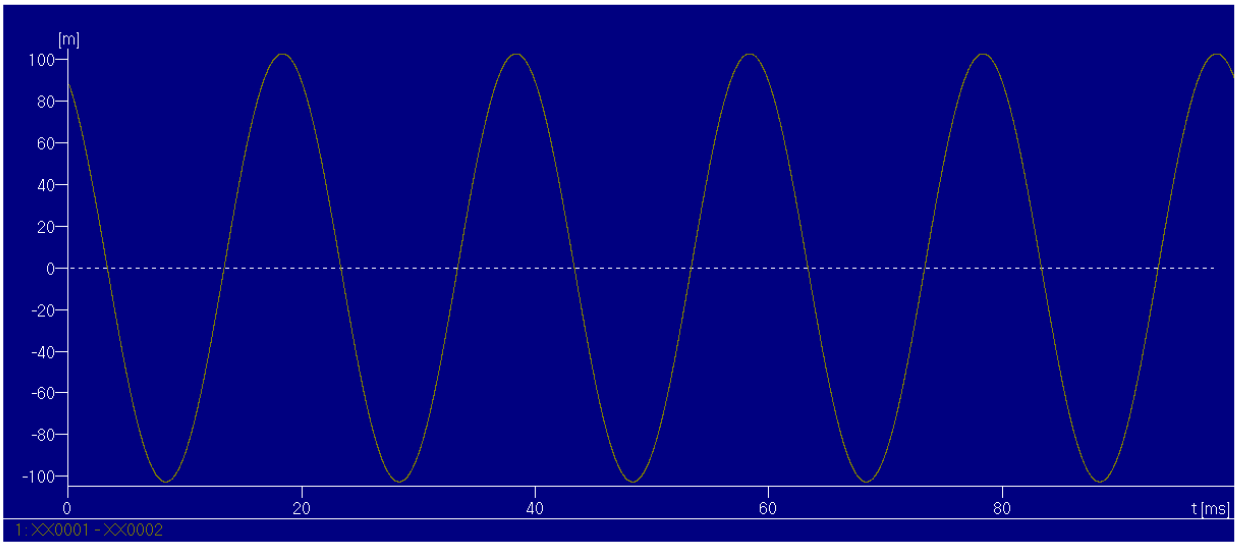

Fig6 – Current I of example 1 simulated with ATP.

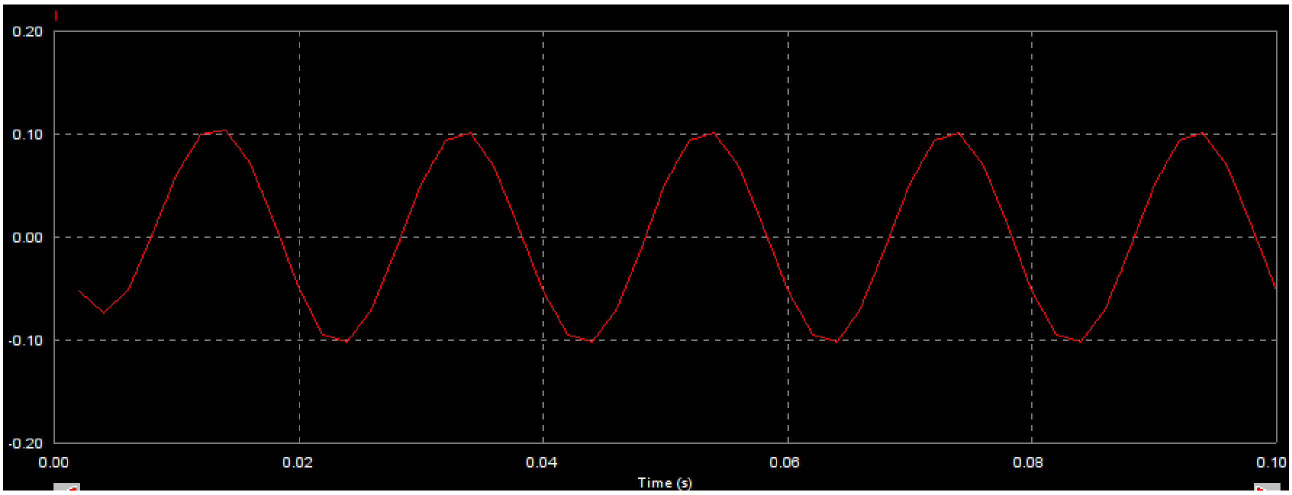

Fig7 – Current I of example 1 simulated with PSIM.

Procedure - example2

In this case another approach to the problem was considered.
Instead of using the same expressions of example 1, the physical laws that describe the behaviour of electric components (theory of circuits) by stating the variables and its derivatives, it was apllied the Laplace Transform to those equations.
That is the most common approach in this type of analysis since it allows the simplification of the expressions by eliminating the derivatives.
It is important to state that it is usual addressing the passive elements of a circuit as impedances[4].
From this, the expression derived for this circuit yields:

$$s^2 i = \frac{1}{L} s v - \frac{R}{L} s i - \frac{1}{LC} i$$

By introducing the Laplace variable, it becomes easier to model the circuit using transfer functions. In this case, we have a second order differential equation, that can be modelled using the Convex module.
Convex module computes the output of linear systems that may be described by a rational transfer function with only real non-positive poles (multiple poles are not allowed but can be implemented as a cascade of Convex modules) of the form:

$$F(s) = \sum_j \frac{A_j}{s - p_j} + \sum_j B_j$$

One possible approach to this problem is the use of block diagram structures.
The simple application of Ohm's law yields:

$$V(s) \Rightarrow \boxed{1/Z(s)} \Rightarrow I(s)$$

Knowing that:

- $V_1(s) = 1 \cdot \sin(w \cdot t)$
- $Z(s) = Z_1(s) + Z_2(s) = 9.23\text{x}10^{-7}s + 3.6\text{x}10^{-5} + 4.138\text{x}10^{-7}s = 3.6\text{x}10^{-5} + 1.341\text{x}10^{-6}s$

then $\frac{1}{Z(s)} = \frac{1}{Z_1(s) + Z_2(s)} = \frac{7{,}481 \times 10^5}{s + 26{,}93}$

In order to implement this with convex, it is necessary to introduce the pair(s) root-coefficient associated to the expressions. In this case, the Xml structure was built as presented below.

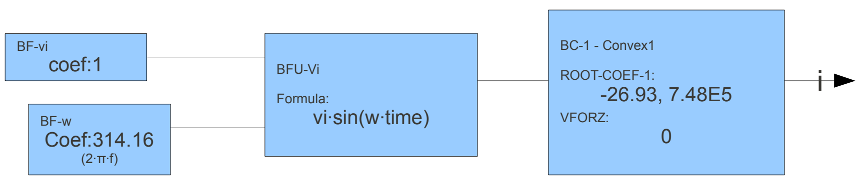

Fig8 – Xml implementation of example 2.

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4 **Impedance** (SI unit: Ω) describes a measure of opposition to alternating current (AC). It extends the concept of resistance to AC circuits, describing not only the relative amplitudes of the voltage and current, but also the relative phases. When the circuit is driven with direct current (DC) there is no distinction between impedance and resistance; the latter can be thought of as impedance with zero phase angle. Impedance is, also, defined as the frequency domain ratio of the voltage to the current.[5]

The circuit was simulated too with Babieca and the results obtained for i were validated both with PSIM and ATP software.

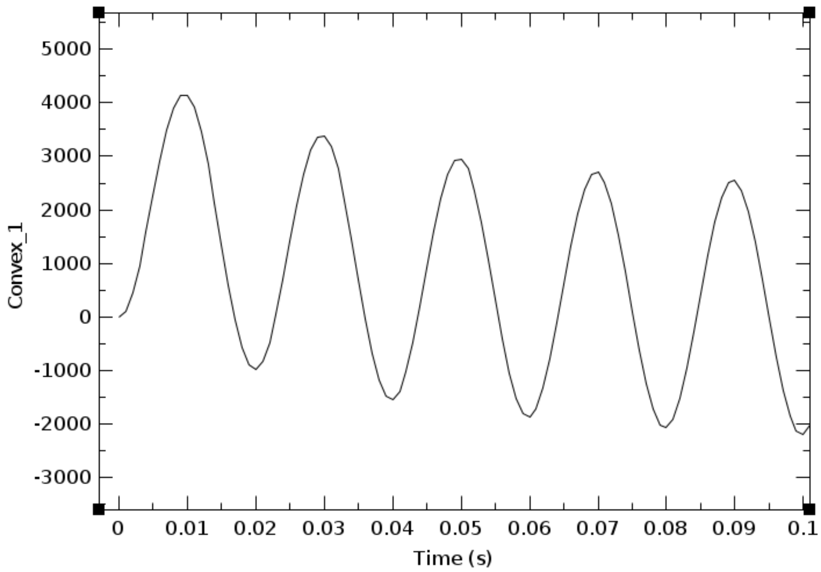

Fig9 – Current I of example 2 simulated with SCAIS.

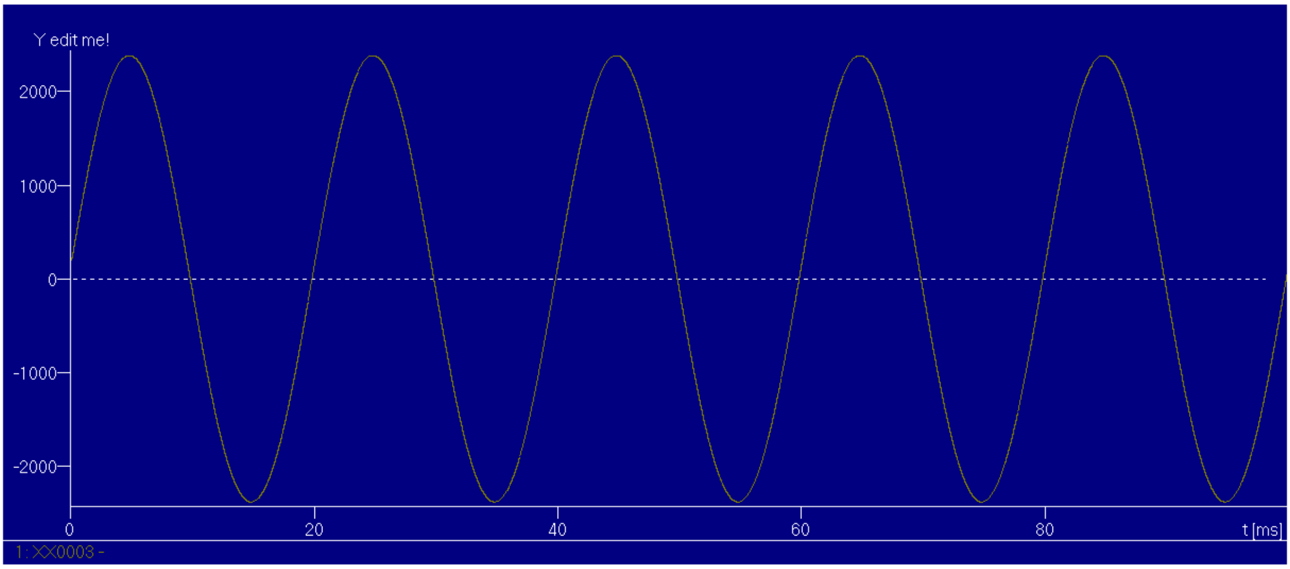

Fig10 – Current I of example 2 simulated with ATP.

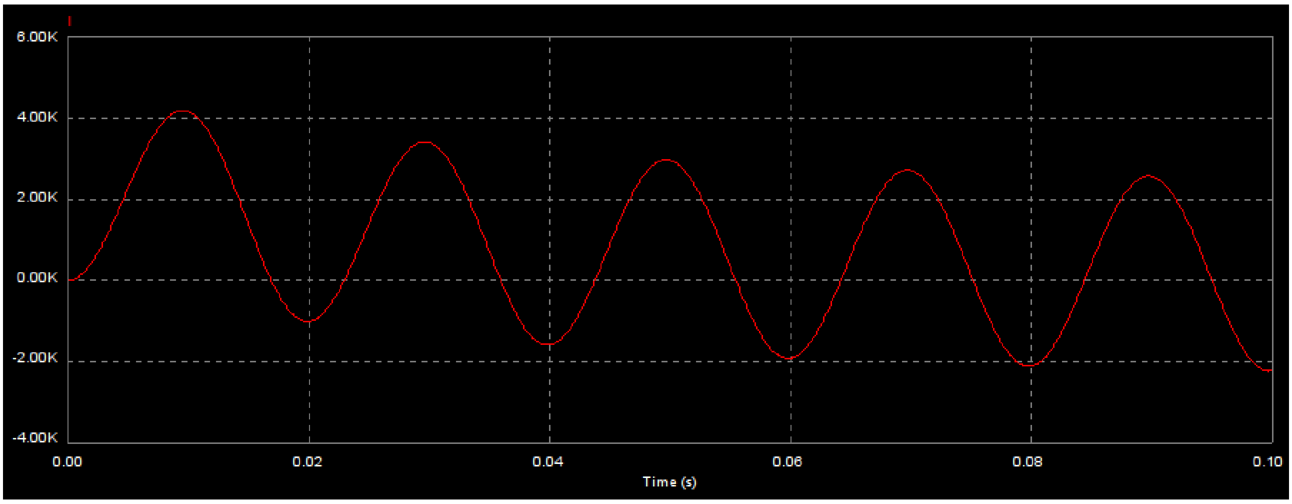

Fig11 – Current I of example 2 simulated with ATP.

Analysis of results

Two simple circuits were implemented through different methods.
Their simulation with SCAIS was successful as it is testified by the results obtained from the two electric circuit simulation programs used, which are congruent.
One note that must be made to the graphics presented above is that the results obtained from ATP (figs.6 and 10) don't simule the initial transient – the initial point for the simulation start is the steady-state regimen.
Despite this, it became clear from other studies that ATP program works quite well for simulating transients.
Therefore, the modelling of a series RL or RLC circuit can be performed by SCAIS and both Vmetodo and Convex have proven able to solve the associated equation systems.

Now, let's take a look at another circuit topology with a slight increase in complexity: a series/paralel RLC circuit:

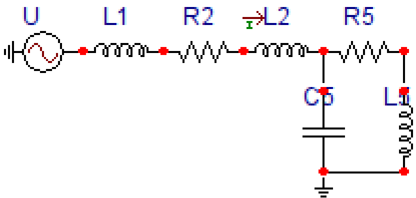

Fig12 – Series/paralel RLC circuit for study.

The circuit presented above was "extracted" from some branches of the one depicted in fig.1 (same variables).
Let's analyze it from the two different perspectives already considered.

I

Using Kirchhoff's voltage law, the circuit can be interpreted as follows.

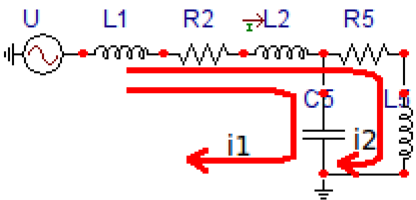

Fig13– Application ofKirchhoff's voltage law for analysis of the Series/paralel RLC circuit (U=V).

This mesh analysis supposes the existance of two currents (i_1 and i_2), each one circulating in one of the paralel branches and both circulating in the L_1-R_2-L_2 branch.
Hereupon we can derive the following expressions:

$$V = L_1\frac{di_1}{dt} + L_1\frac{di_2}{dt} + R_2 i_1 + R_2 i_2 + L_2\frac{di_1}{dt} + L_2\frac{di_2}{dt} + \frac{1}{C_5}\int(i_1\,dt)$$

$$V = L_1\frac{di_1}{dt} + L_1\frac{di_2}{dt} + R_2 i_1 + R_2 i_2 + L_2\frac{di_1}{dt} + L_2\frac{di_2}{dt} + R_5 i_2 + L_5\frac{di_2}{dt}$$

An obvious relation that can be derived from these equations is:

$$\frac{1}{C_5}\int(i_1\,dt) = R_5 i_2 + L_5\frac{di_2}{dt}$$

Integrating and simplifying, these relations yield:

$$i_1 = C_5 R_5 \frac{di_2}{dt} + L_5 \frac{d^2 i_2}{dt^2} \tag{1.1}$$

$$V = L_1 \frac{d(i_1+i_2)}{dt} + R_2(i_1+i_2) + L_2 \frac{d(i_1+i_2)}{dt} + \frac{1}{C_5} \int (i_1\, dt) \tag{1.2}$$

$$V = L_1 \frac{d(i_1+i_2)}{dt} + R_2(i_1+i_2) + L_2 \frac{d(i_1+i_2)}{dt} + R_5 i_2 + L_5 \frac{di_2}{dt} \tag{1.3}$$

When replacing the relation (1.1) in (1.2) and (1.3) it results a differential equation of third degree that isn't of the ordinary type: differential equations with two variables (i_1 and i_2) and their derivatives.

This represents a major dificulty for implementation with the Vmetodo module.

<u>II</u>

Using block diagram logic the first idea to emerge is to use a structure of the form:

$$V(s) \Rightarrow \boxed{1/Z(s)} \Rightarrow I(s)$$

As this topology was adapted from the one in fig.1, we can use the transfer functions already

$$\frac{1}{Z(s)} = \frac{1}{Z_1(s) + Z_2(s) + Z_5(s)}$$

defined:

(series association of Z_1, Z_2 and Z_5)[6].

$$\frac{1}{Z(s)} = \frac{1}{9.23\times10^{-7} s + 3.6\times10^{-5} + 4.138\times10^{-7} s + \dfrac{0.1 + 9\times10^{-3} s}{4.5\times10^{-8} s^2 + 5\times10^{-7} s + 1}}$$

Using a mathematic development software, the expression above may be simplified:

$$\frac{1.66234\times10^{13} + 8.31172\times10^{6}\, s + 748\,055.\, s^2}{1.66294\times10^{12} + 1.49633\times10^{11}\, s + 38.0411\, s^2 + s^3}$$

In order to implement this model with Convex module, it is necessary to determine the associated root-coefficient pairs for the expression. This can be accomplished using any mathematic development software.

Despite that, the determining factor here is that for this “low” level of circuit complexity the expressions obtained hold a considerable level of complexity that constrain its implementation with the SCAIS modules considered in this work.

Conclusions

Some electric circuit topologies were studied in this work with the purpose of applying the ISA methodology to the operation of an electric network with SCAIS.
The objective was to determine whether it was possible or not to model the case-study network and then simulate the protective breakers' switching operations, that impose changes in the circuit topology. The initial idea was that the first part could be implemented with Convex and/or Vmetodo modules and the second by means of setpoints and Logate/LogateHandler modules. Another possible ideia was to define different operation types according to different topologies.

Two examples of electric circuit modelling were presented in detail.
The core of its implementation were the Vmetodo and Convex SCAIS modules.
Even though two types of analysis were made some dificulties emerged that constrained the modelling development.
Convex turned out to be adequate for circuits where the associated transfer functions are easy to calculate (first order circuits, series RLC circuits) as it provided consistent results.
Algebra of transfer functions is not a good method to study networks that change its configuration (topology) over simulation time; also, it requires great expertise to develop the block diagrams' logic.
One conclusion from this work is that the available SCAIS modules don't supply a great range of solutions other than the use of transfer functions.
The dificulties in the determination of those transfer functions when associated to large and complex circuits and consequent implementation with Convex modules turns out to be an impraticable solution.

Nodal-current and mesh-voltage analysis methods are commonly used along with network reduction techniques.
SCAIS' Vmetodo module is able to deal with differential equations until 4^{th} order but, as was demonstrated, the high complexity associated to the diff. equations derived from that type of analysis makes its use impraticable.
In fact, to avoid high complexity in the resulting expressions it resorts to the application of the Laplace Transform. Then, using the properties of complex numbers it becomes much easier to handle the expressions and calculations (s=jw).
Actually, the use of complex numbers – Phasors[5] - is the base of the most nowadays analysis of electric networks, due to the simplicity it entails.
Passive electrical elements are characterized by their impedances: $Z_R=R$; $Z_L=jwL$; $Z_C=1/(jwC)$. Also, electrical quantities such as Voltage an Current can be characterized in polar form: V=|V|·/_φ.
This way it becomes easier the manipulation of expressions and the network analysis can also be made through a matricial way.
Maybe a valuable step towards success in this problem would be the development of a SCAIS module able to handle the representation and numerical operations with complex numbers.

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5 A phasor is a constant complex number, usually expressed in exponential form, representing the complex amplitude (magnitude and phase) of a sinusoidal function of time. Phasors are used by electrical engineers to simplify computations involving sinusoids, where they can often reduce a differential equation problem to an algebraic one. [9]

References

[1] Hortal, J., Izquierdo, J.; Application of the Integrated Safety Assessment Methodology to the protection of electric systems.

[2] Nilsson, J., Riedel, S.; Circuitos eléctricos; 7ª Edición, Pearson Prentice Hall, 2005.

[3] *http://en.wikipedia.org/wiki/Ordinary_differential_equation*

[4] *http://mathworld.wolfram.com/OrdinaryDifferentialEquation.html*

[5] *http://en.wikipedia.org/wiki/Electrical_impedance*

[6] *http://en.wikipedia.org/wiki/Series_and_parallel_circuits*

[7] Beaty, W., Handbook of Electric Power Calculations; 3rd edition, McGraw-Hill, 2001.

[8] Nahvi, M., Edminister, J., Electric Circuits, 4th Edition: Schaum's Outline Series; McGraw-Hill, 2003.

[9] *http://en.wikipedia.org/wiki/Phasor*