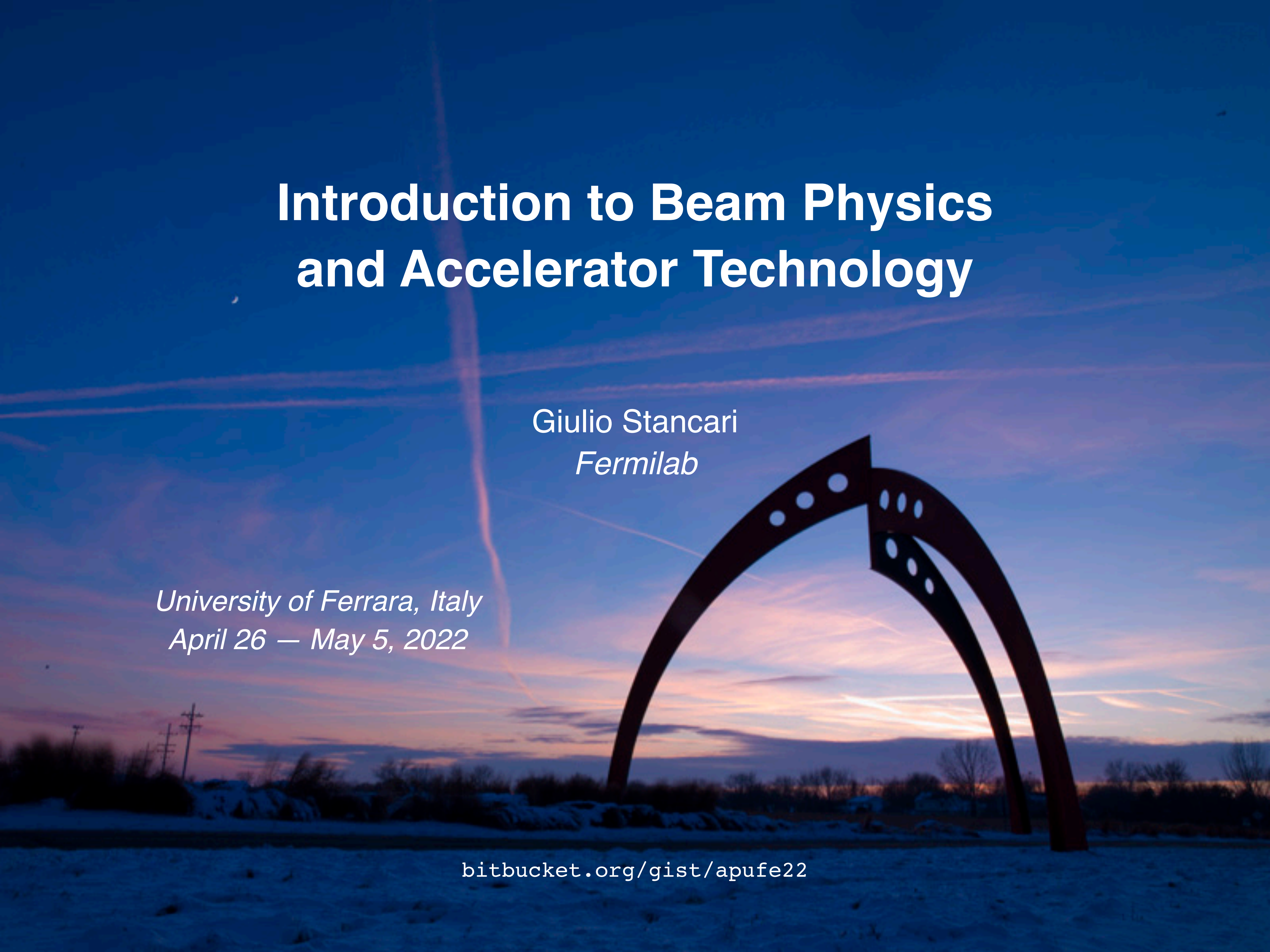


Introduction to Beam Physics and Accelerator Technology

Giulio Stancari

Fermilab

University of Ferrara, Italy

April 26 — May 5, 2022

bitbucket.org/gist/apufe22

Experiment design and luminosity

Experiment design: event rates, cross sections, luminosity

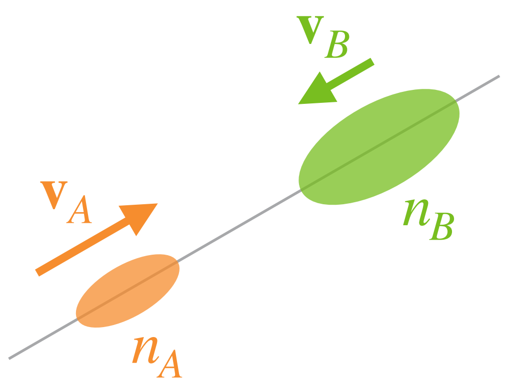
The relationship between **event rate**, **detector efficiency**, **cross section**, **luminosity** and **observation time** is one of the **main factors** in the **design and analysis of experiments**, in both **fixed target** and **collider** configurations

How many events do we expect?

How long will the experiment be?

Will the detector be able to sustain the data rate?

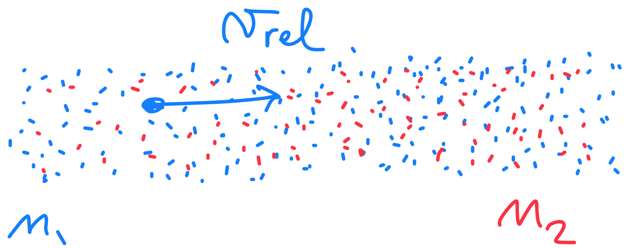
Luminosity quantifies the properties of the interaction: it is the **event rate per unit cross section**



Usually expressed in **cm^{-2} s^{-1}**
Other useful units: **events/μb/s** or **events/fb/yr** etc.

$1 \text{ b (barn)} = 10^{-24} \text{ cm}^2 = 10^{-28} \text{ m}^2$

Event rates in the lab frame (stationary target)



$$\Delta N = \sigma \, v_{\text{rel}} \, n_1 \, n_2 \, \Delta V \, \Delta t$$

- *number of events*: ΔN
- *cross section of the process (invariant by definition)*: σ
- *incoming flux*: $v_{\text{rel}} \, n_1$
- *relative velocity (invariant by definition: velocity of projectiles in the rest frame of the target)*: v_{rel}
- *number densities of projectiles and targets*: n_1, n_2
- *interaction volume*: ΔV
- *interaction time*: Δt

How can we express this relationship in any reference frame?

Particle 4-momenta and relative velocity

4-momenta of particles $p_1^\mu = (E_1, \boldsymbol{p}_1)$ $p_2^\mu = (E_2, \boldsymbol{p}_2)$

The **contraction of the two 4-momenta** is **invariant** — it can be **evaluated in the lab frame**, using the definition of relative velocity

$$p_{1\mu}\, p_2^\mu \equiv E_1 E_2 - \boldsymbol{p}_1 \cdot \boldsymbol{p}_2 = \gamma_1 m_1 m_2 = \frac{m_1 m_2}{\sqrt{1-\beta_{\mathrm{rel}}^2}}$$

The **relative velocity** can therefore be expressed as

$$\beta_{\mathrm{rel}} \equiv \frac{v_{\mathrm{rel}}}{c} = \sqrt{1-\left(\frac{m_1 m_2}{p_{1\mu}\, p_2^\mu}\right)^2} = \ldots = \frac{\sqrt{(\boldsymbol{\beta}_1-\boldsymbol{\beta}_2)^2-(\boldsymbol{\beta}_1\times\boldsymbol{\beta}_2)^2}}{1-\boldsymbol{\beta}_1\cdot\boldsymbol{\beta}_2}$$

(this step requires some algebra)

Generalizing the lab-frame expression for the event rate

invariant (ΔN); *invariant* ($\sigma\, v_{\rm rel}$); *invariant* ($\Delta V\, \Delta t$)

$$\Delta N = \sigma\, v_{\rm rel}\, n_1\, n_2\, \Delta V\, \Delta t$$

densities change like energies under Lorentz transformations (n_1, n_2)

We use the **available 4-vectors** and the **transformation properties of energies and densities** to choose a **factor that makes the whole expression invariant**

$$n_1 n_2 \to n_1 n_2 \frac{p_{1\mu}\, p_2^{\mu}}{E_1 E_2}$$

$$\frac{p_{1\mu}\, p_2^{\mu}}{E_1 E_2} = \frac{E_1 E_2 - \boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{E_1 E_2} = 1 - \boldsymbol{\beta}_1 \cdot \boldsymbol{\beta}_2 \to 1 \quad \text{in the lab frame}$$

Grouping the relative velocity with the new factor, we **define a kinematic factor *v*** with the dimensions of a velocity

$$v \equiv v_{\rm rel}\,(1 - \boldsymbol{\beta}_1 \cdot \boldsymbol{\beta}_2) = \sqrt{(\boldsymbol{v}_1 - \boldsymbol{v}_2)^2 - (\boldsymbol{v}_1 \times \boldsymbol{v}_2 / c)^2}$$

(in general, this is NOT the relative velocity)

General relation, in any reference frame

luminosity

$$\Delta\nu = \epsilon \cdot \sigma \cdot \boxed{v \cdot n_1 \cdot n_2 \cdot \Delta V} \cdot \Delta t$$

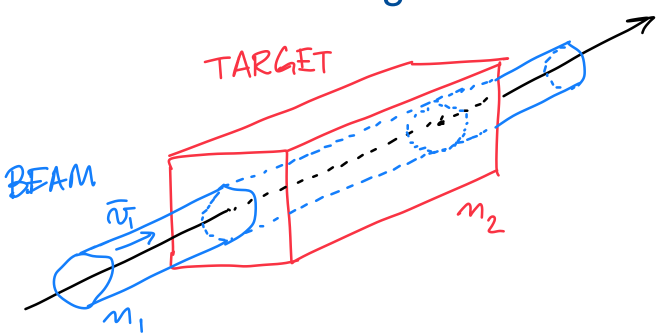
- $\Delta\nu$: number of observed events
- ϵ: detector acceptance and efficiency
- σ: cross section of the process
- v: kinematic factor
- n_1, n_2: number densities
- ΔV: interaction volume
- Δt: interaction time

kinematic factor

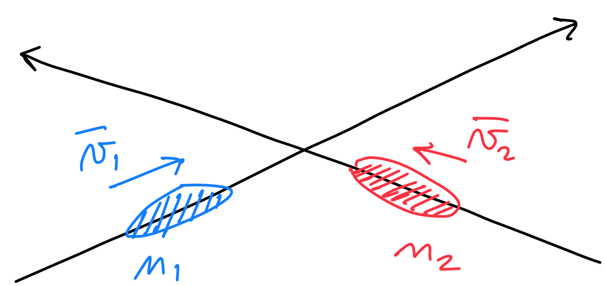
$$v = \sqrt{(\mathbf{v_1} - \mathbf{v_2})^2 - (\mathbf{v_1} \times \mathbf{v_2}/c)^2}$$

Examples:

Fixed target



Colliding beams with crossing angle



Fermilab

Definitions of luminosity

local luminosity

$$v\, n_1\, n_2\, \Delta V$$

luminosity (also: instant luminosity)

$$\mathscr{L}(t) = \int_V v\, n_1\, n_2\, dV$$

overlap integral

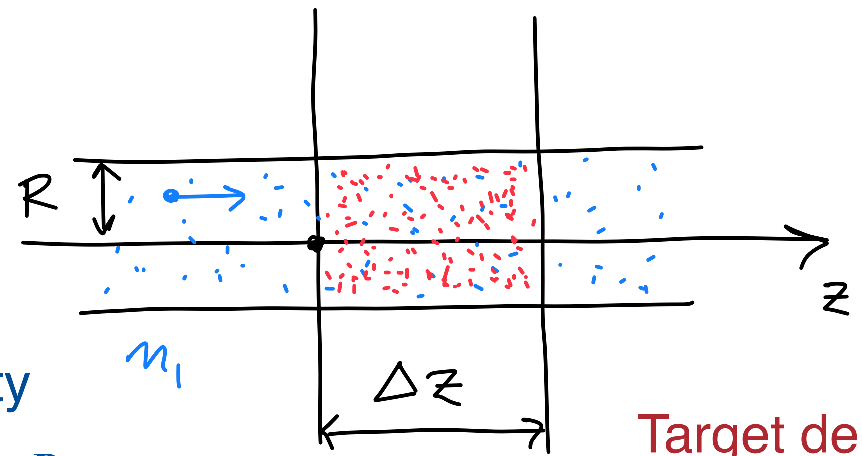
average luminosity over a specified time interval *T*
also: **"instantaneous" luminosity** when averaged over a short time typical of the accelerator (revolution period, pulse period, etc.)

$$\langle \mathscr{L} \rangle_T \equiv \frac{1}{T} \int_T \mathscr{L}\, dt$$

integrated luminosity over a specified time interval *T*,
often a "long" time scale (one operation cycle, one data-taking period, etc.)

$$L = \int_T \mathscr{L}\, dt = \int_T \int_V v\, n_1\, n_2\, dV\, dt$$

Example: continuous and homogeneous beam on fixed target



Beam density

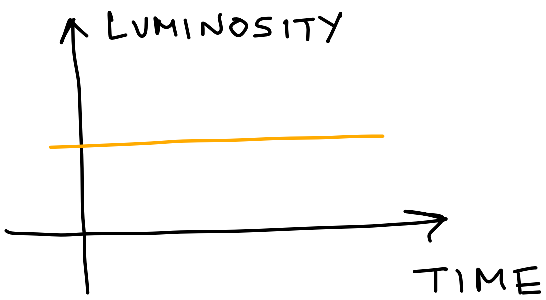
$$n_1 = \begin{cases} n_b & r \le R \\ 0 & R < r \end{cases}$$

Target density

$$n_2 = \begin{cases} n_t & 0 \le z \le \Delta z \\ 0 & \text{elsewhere} \end{cases}$$

Luminosity

$$\mathscr{L} = \int_V v\, n_1\, n_2\, dV = v\, n_b\, n_t\, (\pi R^2)\, \Delta z$$



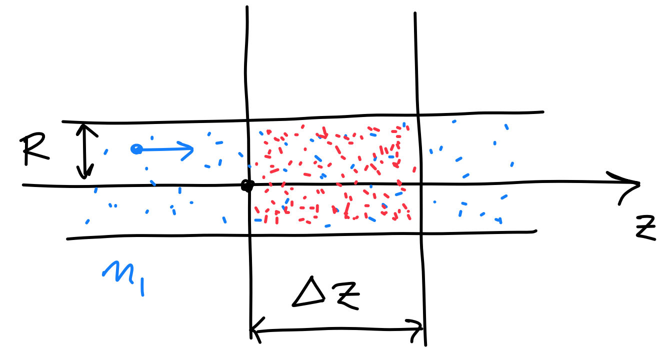
constant in time

Example: continuous and homogeneous beam on fixed target

Numerical example: antiproton-proton collisions in charmonium experiment E-835 at Fermilab. Beam of $N_b = 5 \times 10^{11}$ antiprotons, kinetic energy $T = 5\ \mathrm{GeV}$ distributed approximately uniformly over ring of length $L = 474\ \mathrm{m}$ and transverse beam area A. Gas-jet target with thickness $\Delta z = 0.5\ \mathrm{cm}$ and number density $n_t = 10^{14}\ \mathrm{cm}^{-3}$.

kinematic factor $v = \beta c$

$$\beta = \sqrt{1 - \left(\frac{m}{T+m}\right)^2} = 0.987$$



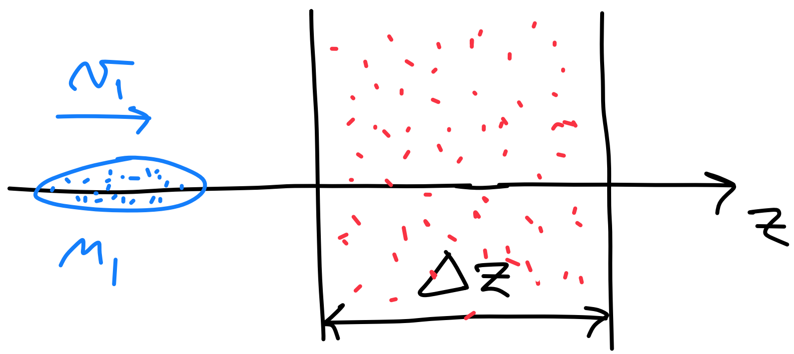
luminosity

$$\mathscr{L} = v\, n_b\, n_t\, A\, \Delta z = v\, \frac{N_b}{LA}\, n_t\, A\, \Delta z = \frac{\beta c\, N_p\, n_t\, \Delta z}{L}$$

$$= 1.56 \times 10^{31}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1} = 15.6\ \mathrm{events}/\mu\mathrm{b/s}$$

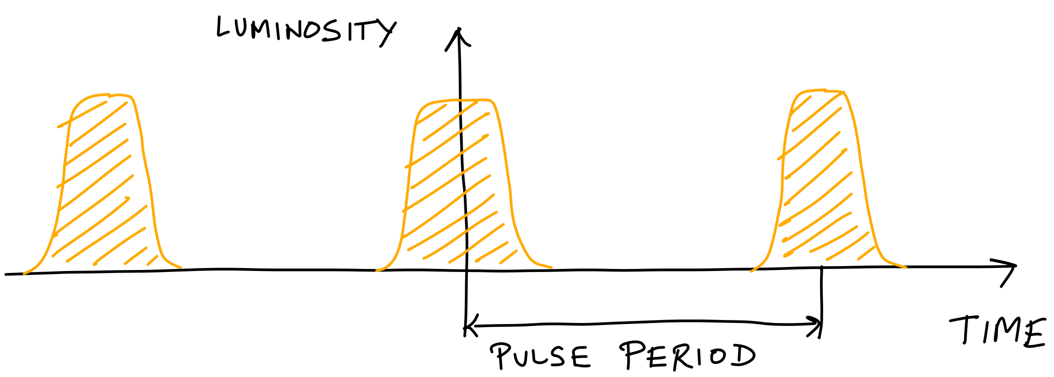
SageMathCell notebook

Example: fixed-target experiment with pulsed Gaussian beam

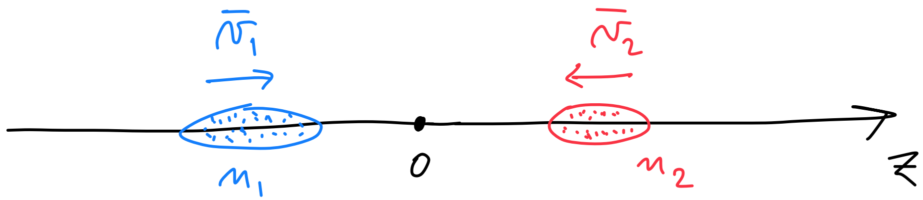


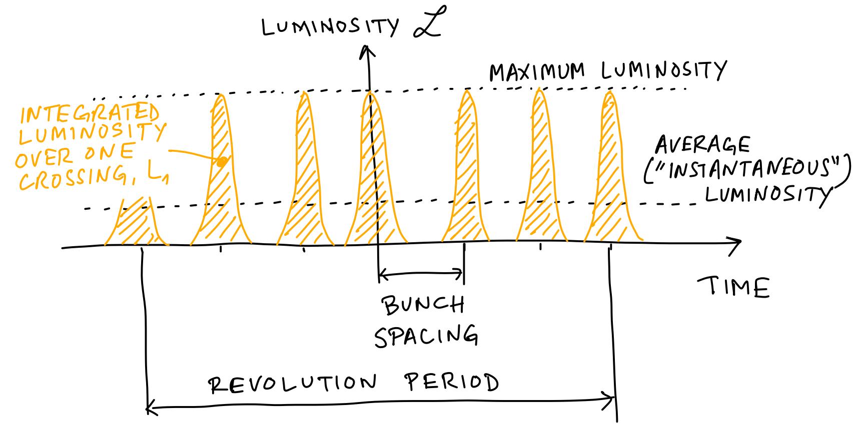
What is the integrated luminosity over one crossing?

What is the average luminosity over many pulses?



Head-on collisions of Gaussian bunches





Counter-propagating Gaussian packets with **equal transverse and longitudinal dimensions**

$$n_1(x,y,z,t) = \frac{N_1}{(2\pi)^{3/2}\,\sigma_x\,\sigma_y\,\sigma_z}\exp\left(-\frac{x^2}{2\sigma_x^2}\right)\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\exp\left(-\frac{(z-vt)^2}{2\sigma_z^2}\right)$$

$$n_2(x,y,z,t) = \frac{N_2}{(2\pi)^{3/2}\,\sigma_x\,\sigma_y\,\sigma_z}\exp\left(-\frac{x^2}{2\sigma_x^2}\right)\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\exp\left(-\frac{(z+vt)^2}{2\sigma_z^2}\right)$$

Peak luminosity at maximum overlap $(t = 0)$

$$\mathscr{L}(0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2v\,n_1(t=0)\,n_2(t=0)\,dx\,dy\,dz$$

$$= \frac{2v\,N_1\,N_2\,\pi^{3/2}\,\sigma_x\,\sigma_y\,\sigma_z}{(2\pi)^3\,\sigma_x^2\,\sigma_y^2\,\sigma_z^2} = \frac{v\,N_1\,N_2}{4\pi^{3/2}\,\sigma_x\,\sigma_y\,\sigma_z}$$

$$\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2}\,dx = \sqrt{2\pi}\,\sigma$$

$$\int_{-\infty}^{\infty} e^{-x^2/\sigma^2}\,dx = \sqrt{\pi}\,\sigma$$

Head-on collision of Gaussian bunches

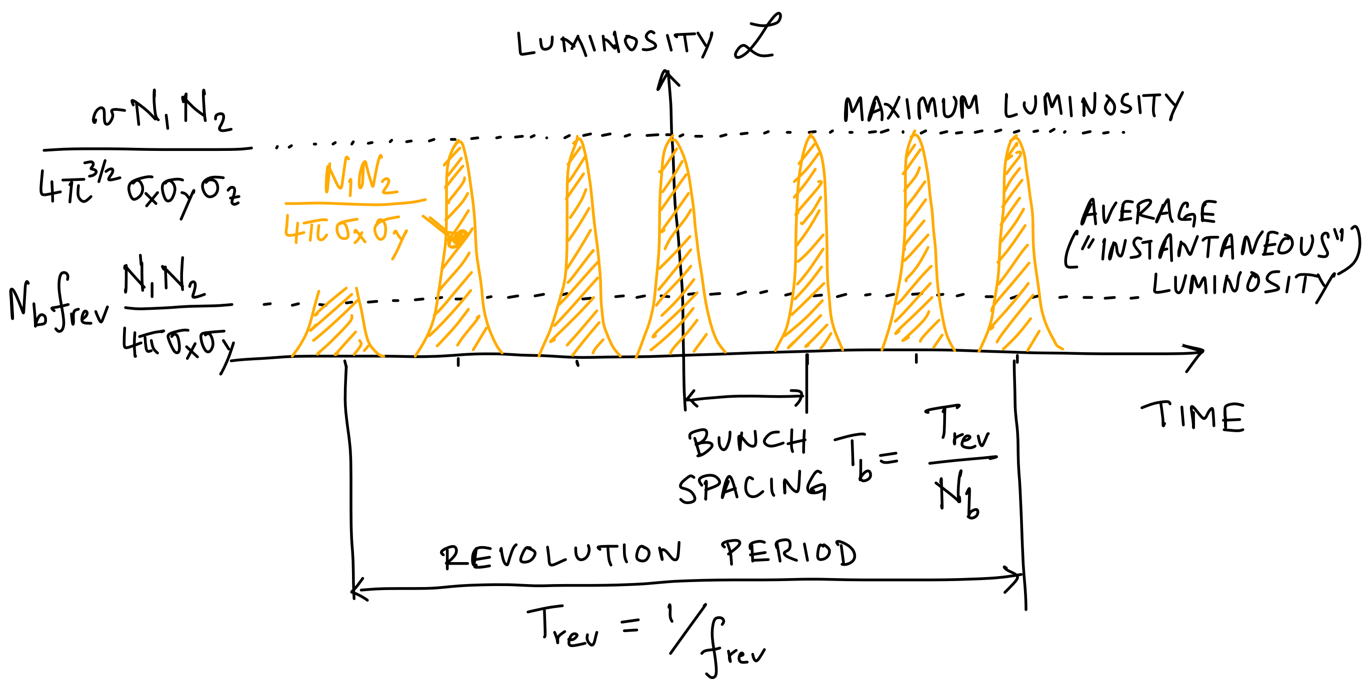
Integrated luminosity over one crossing

$$L_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2v\, n_1\, n_2\, dx\, dy\, dz\, dt = \ldots = \frac{N_1 N_2}{4\pi\, \sigma_x\, \sigma_y}$$

Average ("instantaneous") luminosity

over one revolution

$$\langle \mathscr{L} \rangle = N_b\, f_{\mathrm{rev}}\, \frac{N_1 N_2}{4\pi\, \sigma_x\, \sigma_y}$$



Maximizing collider luminosity

The average ("instantaneous") luminosity in a collider is maximized by

increasing the number of bunches

maximizing bunch populations

$$\langle \mathscr{L} \rangle = N_b \, f_{\mathrm{rev}} \, \frac{N_1 \, N_2}{4\pi \, \sigma_x \, \sigma_y}$$

reducing beam sizes

if possible, increasing the revolution frequency (determined by orbit length and particle speed)

Luminosity optimization gives rise to some of the **challenges** we discussed

Collider luminosity: numerical example

LHC parameters (from the 2021 Review of Particle Properties)

$$\left.\begin{array}{l} C = 26.659 \text{ km} \\ \beta = 1 \end{array}\right. \implies f_{\text{rev}} = ?$$

$$N_1 = N_2 = 1.1 \times 10^{11} \text{ protons/bunch}$$

$$N_b = 2556 \text{ bunches}$$

$$\sigma_x = \sigma_y = 9.5 \ \mu\text{m}$$

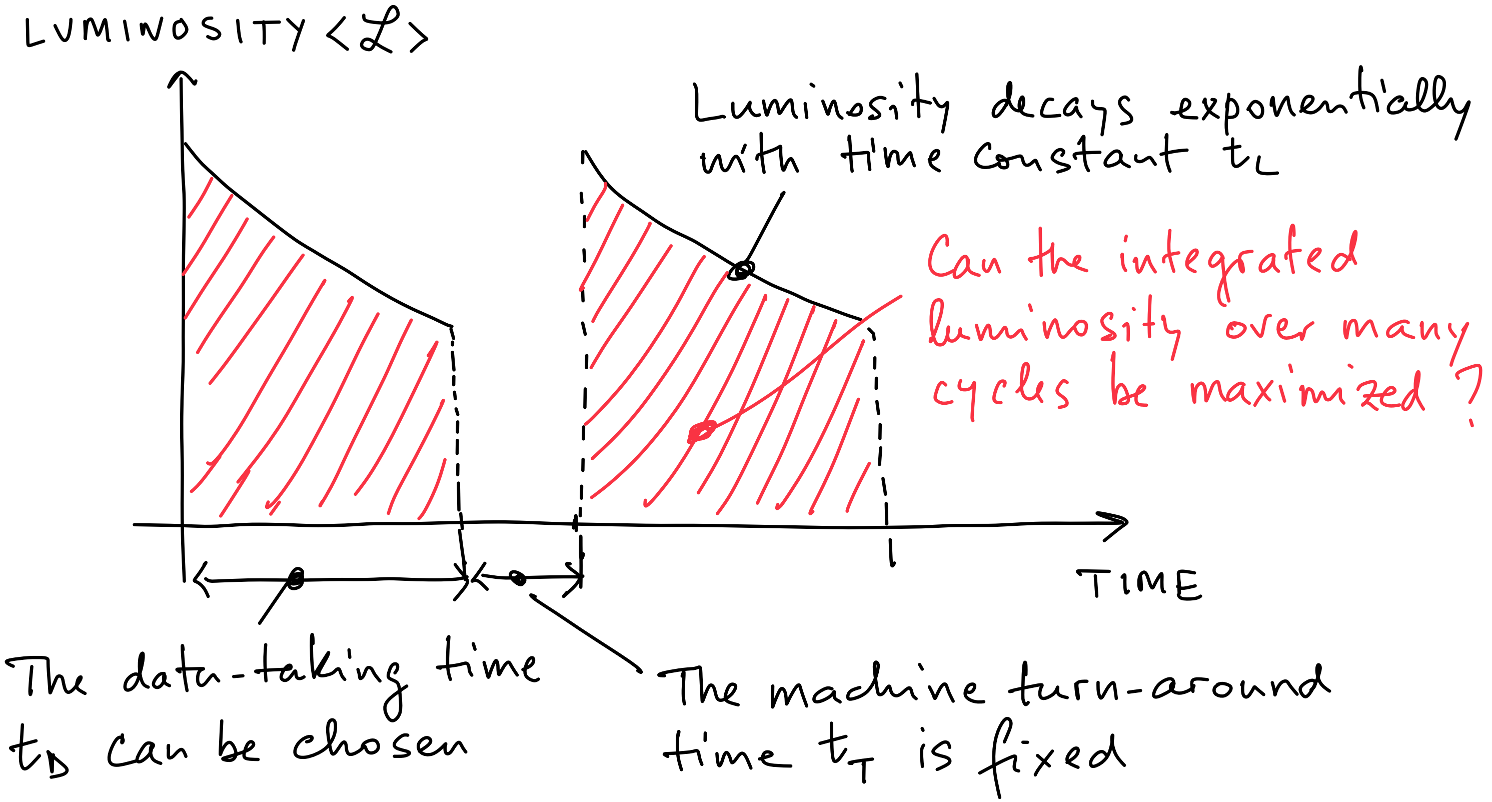
(neglect the crossing angle)

$$\langle \mathscr{L} \rangle = 3.1 \times 10^{34} \text{ cm}^{-2}\,\text{s}^{-1} = 31 \text{ events/nb/s}$$

SageMathCell notebook

Optimization of integrated luminosity

Problem: **maximize integrated luminosity** taking into account **luminosity lifetime** and machine **turn-around time**.
What is the **optimal data-taking duration**?



Observations

Experiment design

What is the optimal spatial distribution of events?

What is the ideal time structure of events?
Constant? Pulsed?

Factors affecting luminosity

emittance dilution

crossing angle

misalignments of the beams

non-zero dispersion

hourglass effect

Measurement of luminosity

absolute vs. relative
known cross sections
van der Meer scans
beam imaging

Grafström and Kozanecki, Luminosity determination at proton colliders,
Prog. Part. Nucl. Phys. 81, 97 (2015)