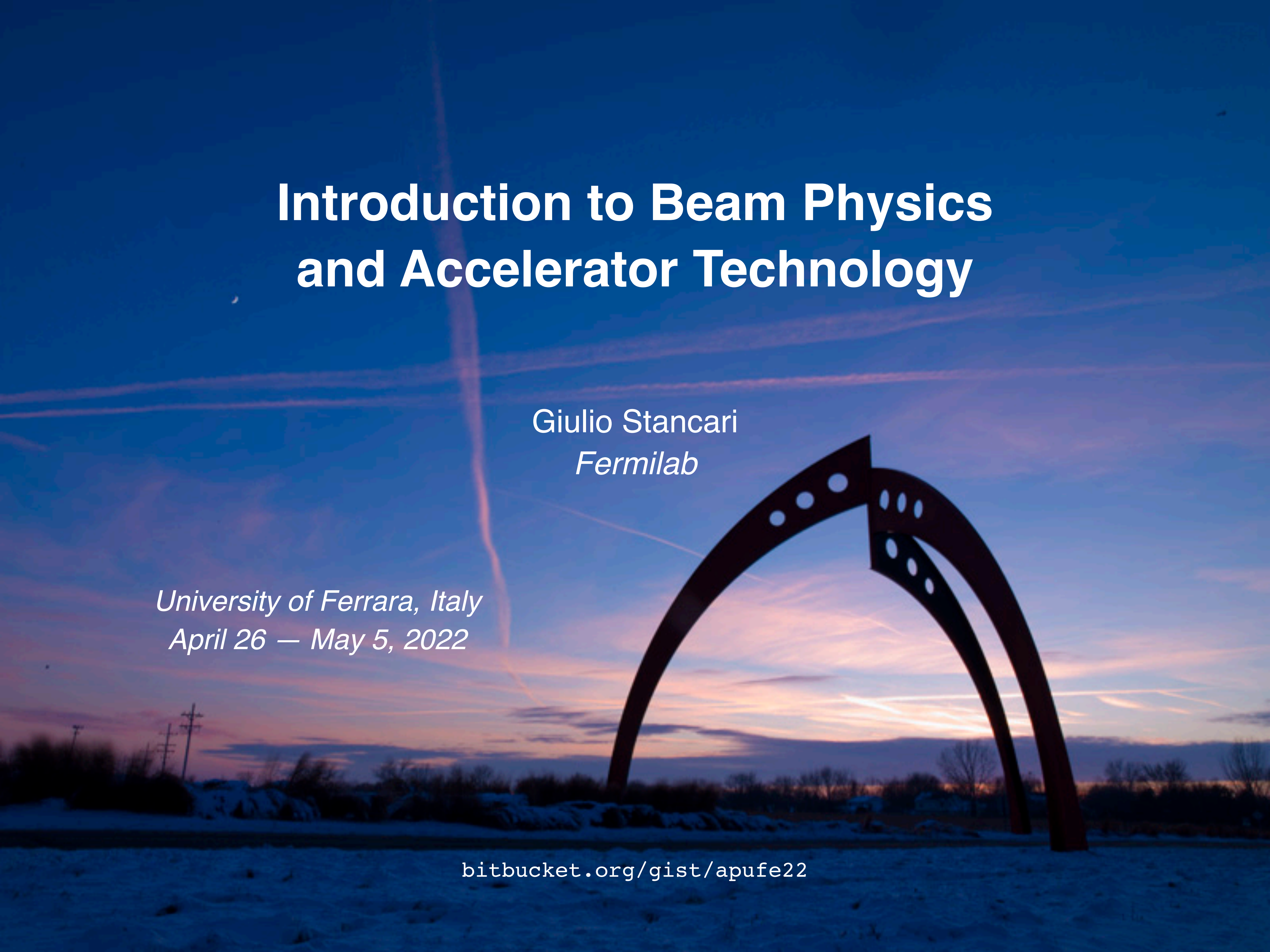
Introduction to Beam Physics and Accelerator Technology

Giulio Stancari
Fermilab

University of Ferrara, Italy
April 26 — May 5, 2022

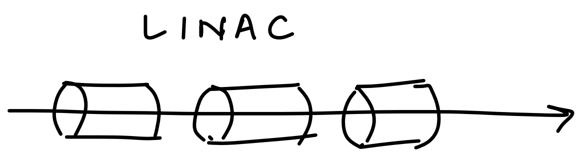
bitbucket.org/gist/apufe22



Longitudinal dynamics and acceleration

Longitudinal dynamics and phase stability

We consider particles **periodically traversing an accelerating station**



Under what conditions **is motion stable**?

(phase stability, synchrotron oscillations, …)

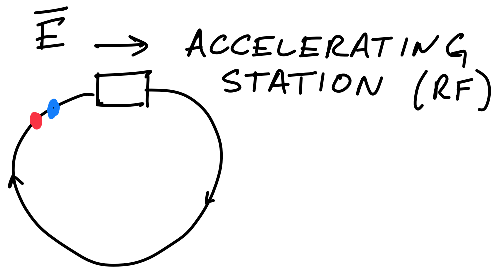
What are the features of this motion?

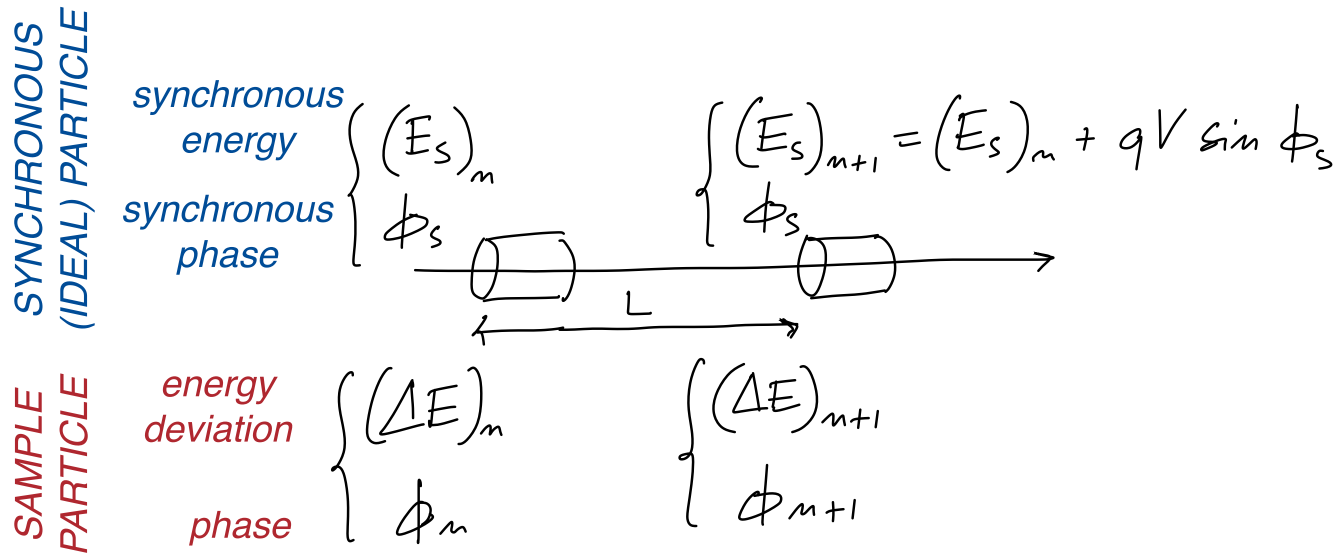
At first, **longitudinal oscillations** can be treated **independently** of **transverse oscillations**. In general, they are **coupled**.

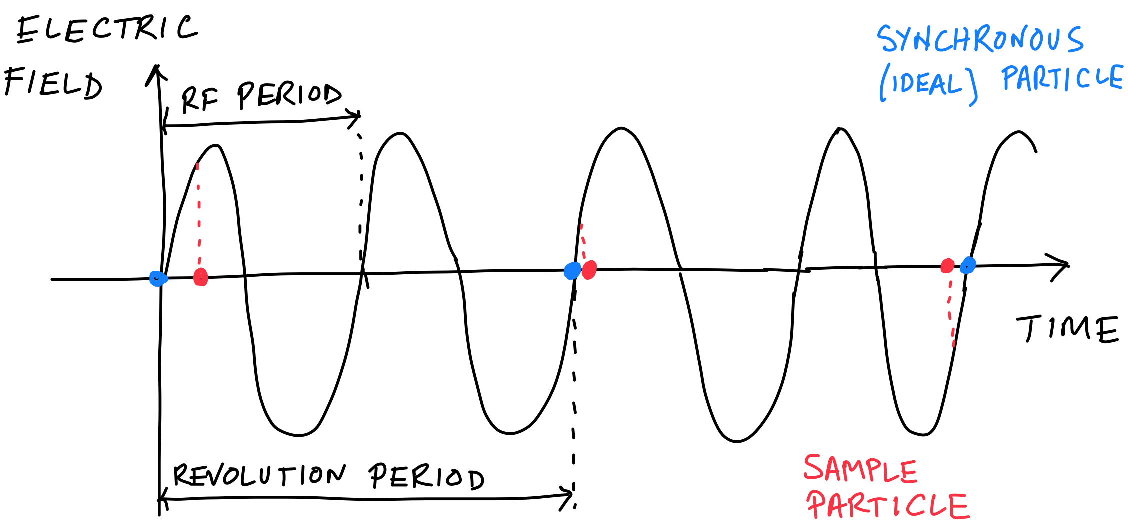
Motion of the sample particle with respect to the ideal particle

oscillating voltage

$$V_{\mathrm{rf}} = V \cdot \sin(\omega_{\mathrm{rf}} t)$$







Choice of **synchronous phase** determines **acceleration**, **deceleration**, or **constant energy**

How do the energy and phase of the sample particle evolve?

How does the revolution period depend on momentum?

period $\tau = \frac{L}{v}$

relative period deviations $\frac{\Delta\tau}{\tau} = \frac{\Delta L}{L} - \frac{\Delta v}{v}$

from relativistic kinematics

$$\frac{\Delta v}{v} = \frac{1}{\gamma^2}\frac{\Delta p}{p}$$

depends on the layout of the machine

$$\frac{\Delta L}{L} \equiv \alpha \frac{\Delta p}{p} \equiv \frac{1}{\gamma_t^2}\frac{\Delta p}{p}$$

momentum compaction (α)

transition factor (γ_t)

transition energy $\gamma_t mc^2$

Therefore

$$\frac{\Delta\tau}{\tau} = \left(\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p} \equiv \eta \frac{\Delta p}{p}$$

phase-slip factor (η)

$\eta < 0$ **below transition** larger momenta result in shorter revolution times (“normal” regime)

$\eta > 0$ **above transition** larger momenta result in longer revolution times (“relativistic” regime)

Evolution of energy and phase deviations

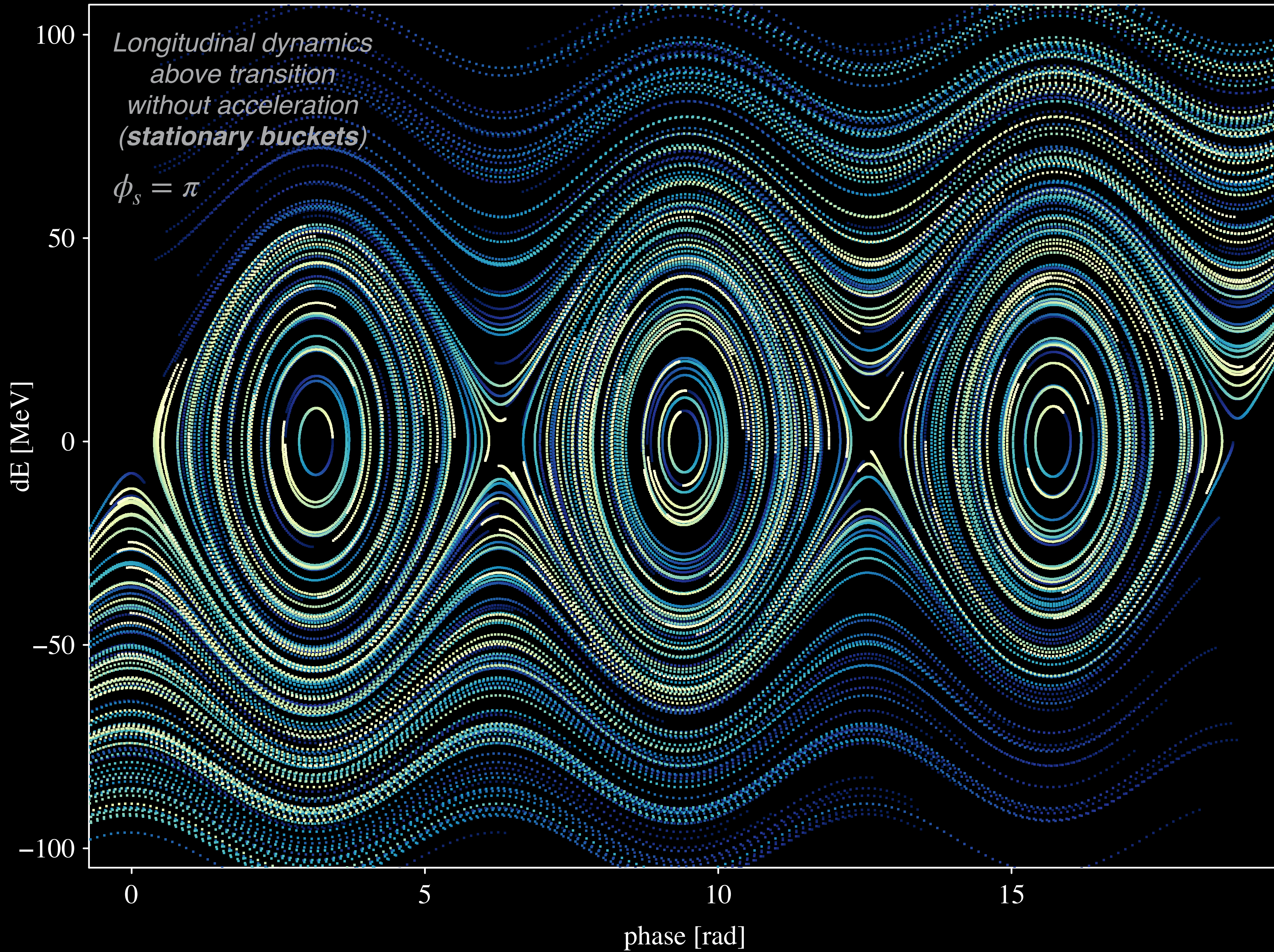
Motion is represented by an **iterated discrete map** (instead of a differential equation) linking **energy deviations** and **phase**

$$\begin{cases} (\Delta E)_{n+1} = (\Delta E)_n + qV(\sin\phi_n - \sin\phi_s) \\ \phi_{n+1} = \phi_n + \dfrac{\omega_{\mathrm{rf}}\tau\eta}{\beta^2 E_s}(\Delta E)_{n+1} \end{cases}$$

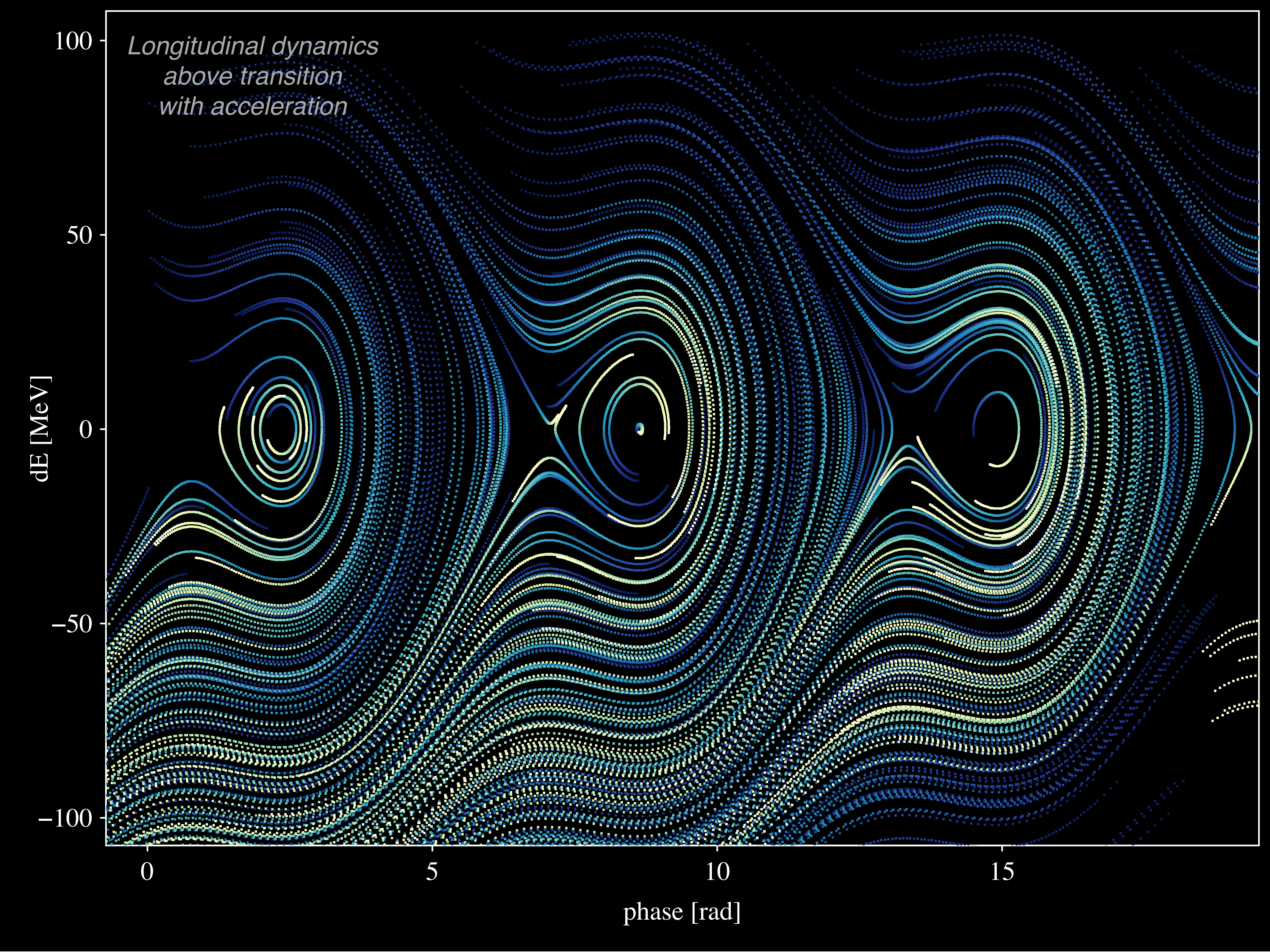
Is motion stable?

There are **stable regions** (“**buckets**”) where particles oscillate around the synchronous particle

[derivations in Edwards and Syphers, Ch. 2]



Longitudinal dynamics
above transition
without acceleration
(**stationary buckets**)
$\phi_s = \pi$
100
50
0
−50
−100
dE [MeV]
0
5
10
15
phase [rad]



Longitudinal dynamics
above transition
with acceleration
100
50
0
−50
−100
dE [MeV]
0
5
10
15
phase [rad]

Continuous approximation of discrete map for small oscillations

$\Delta\phi \equiv \phi - \phi_s$

n continuous

$$\frac{d^2(\Delta\phi)}{dn^2} + \left(-\frac{\omega_{\rm rf}\tau\eta qV\cos\phi_s}{\beta^2 E_s}\right)(\Delta\phi) = 0$$

$\eta \cdot \cos\phi_s < 0$ for stability

Harmonic oscillator with frequencies

$$\nu_s \equiv \frac{1}{2\pi}\sqrt{-\frac{\omega_{\rm rf}\tau\eta qV\cos\phi_s}{\beta^2 E_s}}$$

synchrotron tune

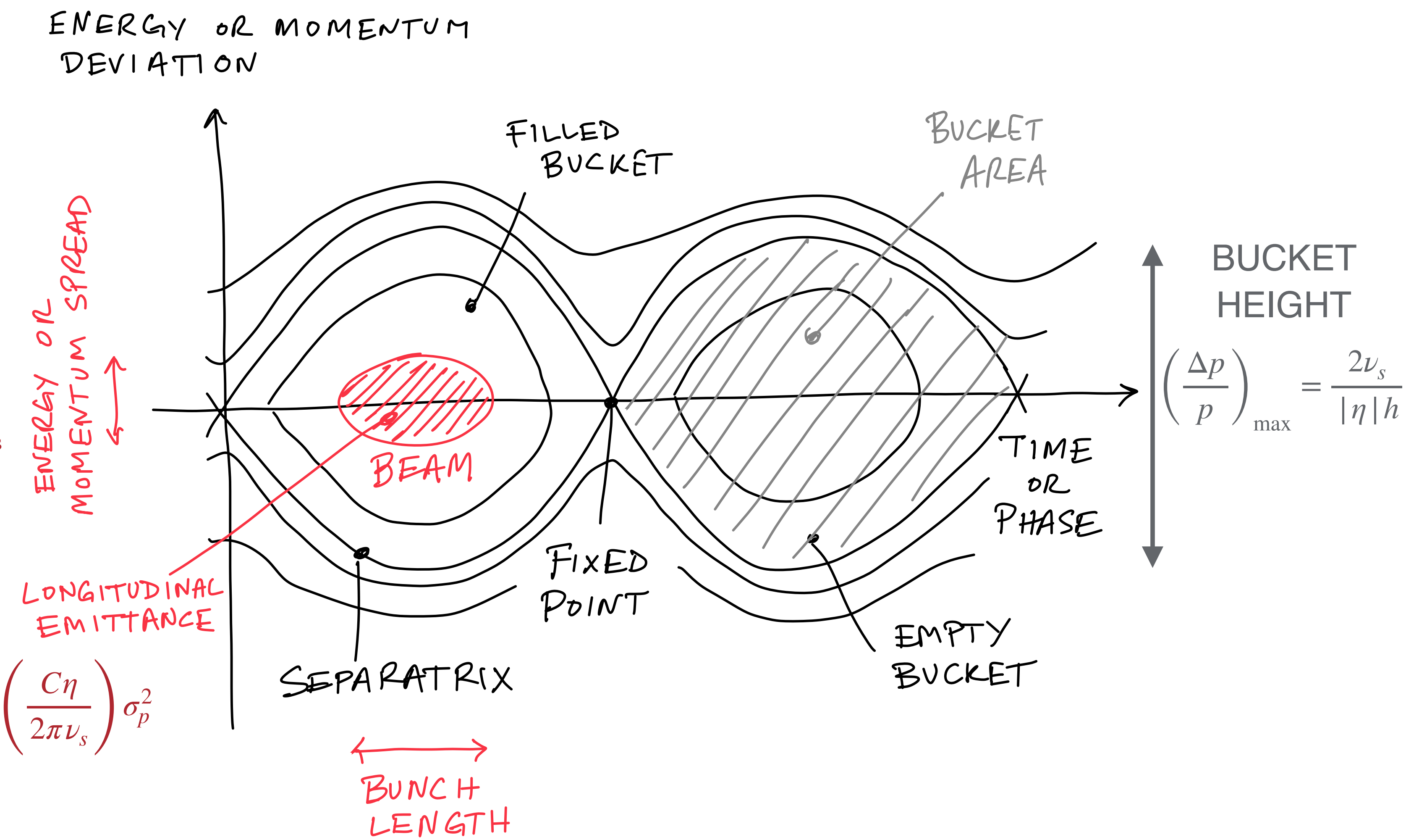
in number of turns

$$f_s \equiv \frac{\nu_s}{\tau}$$

synchrotron frequency

in terms of time^{-1}

Topology and features of longitudinal phase space



$$\sigma_p \equiv \left(\frac{\Delta p}{p}\right)_{\rm rms}$$

$$\left(\frac{\Delta p}{p}\right)_{\max} = \frac{2\nu_s}{|\eta|h}$$

$$\epsilon_z = \left(\frac{C\eta}{2\pi\nu_s}\right)\sigma_p^2$$

$$\sigma_z = \left(\frac{C\eta}{2\pi\nu_s}\right)\sigma_p$$

Observations

- At the **transition energy** ($\eta = 0$) there are **no phase oscillations** — motion is “frozen”
- **Linacs** always operate **below transition**. As the energy increases, they approach transition and constant phases.
- The **discrete map of energies and phases is non-linear**. **Oscillation frequencies** are not constant — they **decrease with amplitude**