PowerGIM

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1 Introduction

Power Grid Investment Model (PowerGIM) was originally a Transmission Expansion Planning (TEP) model incorporated into a market model known by the name PowerGAMA. PowerGAMA is a deterministic LP optimization problem, while PowerGIM is formulated as a MILP to incorporate binary and integer investment variables. PowerGIM can be considered as an Expasion Planning model for both intrastucture- and generator investments. Moreover, it has the ability to account for uncertainty since it is formulated as a Two-stage Stochastic Program with variables related to investment decisions in the first stage, and operational variables in the second stage.

The model is implemented with Pyomo [1, 2], where the stochastic features are enabled with the PySP package [3]. Pyomo includes a collection of Python software packages that supports a diverse set of optimization capabilities for formulating and analyzing optimization models. Although most AMLs are implemented in custom modeling languages, Pyomo's modeling objects are embedded within Python, a full-featured high-level programming language that contains a rich set of supporting libraries.

PySP leverages the fact that Pyomo's modeling objects are embedded within a full-featured high-level programming language, which allows for transparent parallelization of subproblems using Python parallel communication libraries.

2 About Pyomo and PySP

The nice part with Pyomo and PySP is that it allows you to formulate your stochastic model as a deterministic model, but you will have to include a scenario tree with representative scenario datasets in order for Pyomo to generalize a stochastic problem. In short, you will need to provide the following files:

- MyModel.py (deterministic formulation of your optimization problem)
- MyScenarioStructure.dat (equivalent to a scenario tree, where you define the stage wise dependence and probabilities)

- MyScenarioData1 (data input for scenario 1)
- ...
- MyScenarioDataX (data input for scenario X)

Note that PowerGIM is structured in a bit more elegant way, in order to be more user-friendly, but the aforementioned files will still give a good idea about what is required in order to build a successful optimization program.

2.1 Pyomo documentation

Download Pyomo: http://www.pyomo.org/

For more functionality within Pyomo ans PySP, please consult the online documentation: https://software.sandia.gov/downloads/pub/pyomo/PyomoOnlineDocs.html

2.2 Modelling Syntax

In your model file, import pyomo

```
from pyomo.environ import *
```

Which imports the full Pyomo environment. Formulate your model as deterministic, but remember to separate stage-dependent costs in your objective function, e.g.:

```
def Total_Cost_Objective_rule(model):
            return model.FirstStageCost + model.SecondStageCost
model.Total_Cost_Objective = Objective(rule=Total_Cost_Objective_rule, sense=minimize)
```

2.2.1 Deterministic solution

If you want to solve the model as a deterministic MILP, you can do it by scripting, e.g.:

```
opt = SolverFactory("cplex")
instance = model.clone()
results = opt.solve(instance,
                        tee=True, #stream the solver output
                        keepfiles=False, #print the LP file for examination
                        symbolic_solver_labels=False) # use human readable names

instance.solutions.load_from(results)
print('First stage costs: ', value(instance.FirstStageCost)/10**9, 'bnEUR')
print('Second stage costs: ', value(instance.SecondStageCost)/10**9, 'bnEUR')
```

2.2.2 Stochastic solution

If you want to solve it as a stochastic program, you could use a command window to execute the model script together with a scenario structure. To make it easier for us who are most familiar with scripting, you can include the scenarios already in your model file by using a callback function, e.g.:

```
s1 = pd.read_excel('scenarios/Storage_1.xlsx', sheetname=gen)
s2 = pd.read_excel('scenarios/Storage_2.xlsx', sheetname=gen)
s3 = pd.read_excel('scenarios/Storage_3.xlsx', sheetname=gen)
s4 = pd.read_excel('scenarios/Storage_4.xlsx', sheetname=gen)
genMax = {}
genMax['Scenario1'] = s1['max'].to_dict()
genMax['Scenario2'] = s2['max'].to_dict()
genMax['Scenario3'] = s3['max'].to_dict()
genMax['Scenario4'] = s4['max'].to_dict()

def pysp_instance_creation_callback(scenario_name, node_names):
    instance = model.clone()
    instance.demandMax.store_values(demandMax[scenario_name])
    instance.genMax.store_values(genMax[scenario_name])
    return instance
```

Then you have initiated all the scenario input data, and you could type the following in a command window:

Solving with progressive hedging

```
runph -m PwrGSIM.py -i data\ScenarioStructure.dat --solver=cplex
  --default-rho=0.9
```

Solving with deterministic equivalent

```
runef -m PwrGSIM.py -i data\ScenarioStructure.dat --solver=cplex
  --solve
```

Where –m is short for *model-directory*, and –i is short for *instance directory*. When solving with progressive hedging algorithm, you will have to define a rho. Those commands could be included in a run-file (.bat).

3 Power System Modelling

Modelling of the physical power flows in an optimization model is a difficult task, mainly due to the different laws and characteristics apply to commercial and physical exchange of electricity in an interconnected system [4]. According to Kirchhoff's circuit laws, physical power flows may take multiple paths though a transmission grid [5]. The following sections will discuss alternative ways to model power flows in an electricity system, starting with a classical *transport model* approach followed by a more sophisticated *load flow* calculation.

3.1 NTC Capacity Allocation

The net transfer capacity (NTC), or rather the available transfer capacity (ATC) obtained when subtracting the already allocated capacity (AAC), is the maximum allowed commercial exchange between two adjacent bidding areas that complies with the security standards of the given synchronous area, and takes into account the technical uncertainties on future grid conditions [6]. These limits are determined by the Transmission System Operators (TSOs) to facilitate the market transactions while safeguarding the grid.

The NTC is defined as total transfer capacity (TTC) less the transmission reliability margin (TRM) [7]. The TRM is a part of the total capacity that is withheld from the market by the TSO in order to manage possible congestions and the physical flows, including transit flows, that will occur in the interconnected system. The transit flows are not taken explicitly into account in the NTC market clearing, also known as coordinated net transfer capacity (CNTC). As a result of this, inefficient allocation of the total capacity might occur if the allocated TRMs are not fully utilized. As transit flows are hard to predict, capacity calculation in an interconnected grid becomes complex and might lead to suboptimal or inefficient capacity allocations, as the transmission constraints in the market clearing algorithm are given as NTCs [8].

$$\mathrm{ATC} = \mathrm{NTC} - \mathrm{AAC} = (\mathrm{TTC} - \mathrm{TRM}) - \mathrm{AAC} \tag{1}$$

The equivalent setup would be used in expansion planning models using a transport formulation of the grid, i.e. only limiting the flow through a branch. The next subsection will include the physical nature of loop-flows, meaning that power flows are not guaranteed to flow directly from A to B since the power flow will depend on "the path of least resistance".

3.2 FB Capacity Allocation

As the entire power system is physically interconnected, an action in one part of the system will in principle affect the entire system, in the form of transit flows, also known as loop flows. This interdependency can be expressed through load flow equations or a power transfer distribution factor (PTDF) matrix. Incorporation of the aforementioned will result in what we refer to as a flow-based (FB) market clearing.

However, in contrast to NTC, capacity allocation is no longer a choice of the TSO that is made in advance, but it is an outcome of the market clearing. Hence the allocation is market driven, creating a stronger connection between the power markets and the physical system [7]. For this reason, FB market clearing is the preferred approach in the Network Code on Capacity Calculation and Congestion Management (NC CACM) developed by the ENTSO-E [9], stating that a FB approach should be used unless its added value can be disproved compared to an NTC approach [8].

The use of a flow-based model allows for a more precise modelling of the physical flows, as the constraints of the FB optimization problem are simplified grid models, reflecting the impact of changing net positions on the flows in the network [7]. This leads to a more efficient capacity allocation as the market takes all flows in the system into account and no transfer capacity has to be withheld from the market. Transit flows can then be monitored and possible congestions are taken care of in the market clearing algorithm directly [8]. Additionally, the use of PTDFs provide the opportunity of a single allocation mechanism including a mixture of AC and DC elements, often referred to as hybrid coupling [8].

3.2.1 Power Flow Equations

A non-linear set of equations can be used to describe the steady-stage relationship between active and reactive power injections, and voltages. These equations reflect the true physics behind electric power flows always following the path of least resistance, in accordance with Kirchhoff's circuit laws.

$$P_i = U_i \sum_{k=1}^{N} U_k (G_{ik} cos(\delta_i - \delta_k) + B_{ik} sin(\delta_i - \delta_k)) \tag{2}$$

$$Q_i = U_i \sum_{k=1}^{N} U_k (G_{ik} sin(\delta_i - \delta_k) + B_{ik} cos(\delta_i - \delta_k)) \tag{3}$$

Large-scale optimization programs should preferably be linear and convex, which is not the case for the *AC power flow* or *load flow* equations. One way to liearize the non-linear power flow relations is to assume the following:

1) The resistance R has a way lower magnitude than the reactance X. A typical ratio between 2 - 10 is not uncommon.

$$R \ll X \Rightarrow Y = G + jB = \frac{1}{Z} = \frac{1}{R + jX} \approx \frac{1}{jX} \Rightarrow G \approx 0 \text{ and } B \approx -\frac{1}{X} \tag{4}$$

2) All voltages are approximately the same in terms of their reference value in *per unit*. A rage of 0.95 - 1.05 is normal. Unity is therefore a good approximation.

$$U_i = U_k = 1 \qquad \forall k \in N \tag{5}$$

3) Two adjacent nodes have approximately the same voltage angle, where the difference is usually very small ranging from 0 - 15 degrees (or 0 - 0.26 radians). Since $cos(0) = 1$ we can approximate the small angle deviations in the cosine terms to one. While small deviations in the sine terms could be replaced by the angle difference itself.

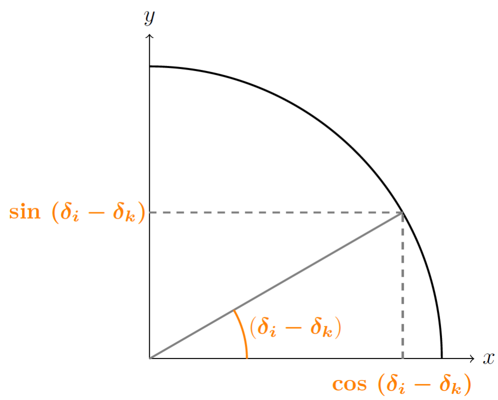

Figure 1: Trigonometric relation of active and reactive power (or phase angle between voltage and current) [10].

$$cos(\delta_i - \delta_k) \approx 1 \tag{6}$$

$$sin(\delta_i - \delta_k) \approx \delta_i - \delta_k \tag{7}$$

These three approximations will result in the following linear set of AC power flow equations:

$$P_i = \sum_{k=1}^{N} B_{ik} sin(\delta_i - \delta_k) \tag{8}$$

$$Q_i = \sum_{k=1}^{N} -B_{ik} \tag{9}$$

Note that the expression for reactive power is constant and will therefore not be a variable in the power flow calculations. Hence, we can neglect this equation, so $Q_i = Q_{ij} = 0$. The resulting power flow equations, i.e. (2), is often referred to as *DC load flow equations*. However, although the name could indicate a that it describes DC currents and voltages, it should be noted that it as a linearization of AC power flow equations. The matrix notation of DC load flow equations are given below:

$$\mathbf{P} = \mathbf{B}\delta = \mathbf{Y_{bus}}\delta \tag{10}$$

$$\delta = \mathbf{Y_{bus}^{-1}}\mathbf{P} = \mathbf{Z_{bus}}\mathbf{P} \tag{11}$$

Where the bus admittance matrix is given by the entries:

$$Y_{bus,ii} = \sum_{k \in A_i} \frac{1}{x_{ik}} \tag{12}$$

$$Y_{bus,ik} = -\frac{1}{x_{ik}} \tag{13}$$

where A_i is all adjacent nodes to node i.

An infinite number of solutions would be found for Equation (10) - (11) due to the interdependency in the equations, and the Y_{bus}-matrix is said to be singular. A reference point has to be given in order to get one unique solution. One solution is to pick a *reference node*[1], where the voltage angle is set to zero degrees or radians. The augmented matrix is denoted $Y_{bus}^{'}$ where a reference node is chosen. E.g., if node one is chosen, row one and column one is deleted from the original Y_{bus} matrix, or one is added to the corresponding diagonal element $(1,1)$.

$$\mathbf{P} = \mathbf{B}\delta = \mathbf{Y_{bus}^{'}}\delta \tag{14}$$

3.2.2 Power Flow Matrix

The power flow equations can be translated into a flow-matrix, known as PTDF matrix. For a given net-position at a node, the distributional flow patterns can be recognized with the PTDF matrix. Recall the flow between two nodes:

$$P_{ik} = B_{ik}(\delta_i - \delta_k) \tag{15}$$

The DC load flow equations are used to derive the PTDF matrix. The power transmission distribution factors (PTDFs) are sensitivity factors expressing the percentage of one unit export from a given node, or an area as described later, that will flow on a particular line. In other words, the *change* in power flow on a given line as expressed by Equation 18 where the prime indicates the augmented values with a reference point to account for the singularity of the bus impedance matrix.

if we assume additional power ΔP_i is injected into node one, the change in voltage angles will be:

$$\Delta\delta_i = Z_{bus,ii}^{'}\Delta P_i \tag{16}$$

$$\Delta\delta_k = Z_{bus,ki}^{'}\Delta P_i \tag{17}$$

the change in active power flow on a branch can be formulated as:

$$\Delta P_{ik} = B_{ik}(\Delta\delta_i - \Delta\delta_k) = B_{ik}\Delta P_n(Z'_{bus,ii} - Z'_{bus,ki}) \tag{18}$$

[1] A node in a power system known as slack bus, swing bus, or reference bus/node. It should be chosen where the voltage magnitude and angle is specified. This node will make it possible for the system equations to get one unique solution that incorporates total power injection, losses, and loads.

If ΔP_n is set to unity, the effect on power flow on line i-k can be regarded as the PTDF for that line per unit net power injection in node n. This is commonly referred to as the sensitivity factor, or power transmission distribution factor for line i-k arising from the net position (NP) of node n, denoted $PTDF_{ik,n}$.

$$PTDF_{ik,n} = B_{ik}(Z'_{bus,in} - Z'_{bus,kn}) \tag{19}$$

However, as price calculations are done on an area level, corresponding aggregated area PTDFs has to be created. In the market clearing algorithm, only connections between bidding areas, referred to as the critical network elements (CNEs), are taken into account. Area-to-CNE PTDFs indicates how a change in the aggregated net position in an area affects the flow on a given CNE [8]. As the NP of all nodes in an area influence the flow on a given CNE to varying degree, incorrect weighting of a node could yield inaccurate estimates of the actual flows on a CNE. One way to cope with this problem, is the use of Generator Shift Keys (GSKs), describing the effect a change in net position of a node has on its area's net position. Different strategies defines how the node-to-line PTDFs should be weighted in accordance to each other, in order to obtain equivalent area-to-line PTDFs [8]. Gebrekiros *et al.* [11] presents three different schemes with varying degree of complexity and information requirement.

A generic formulation of the PTDF of CNE i-k arising from the net position of area A, denoted $PTDF_{ik,A}$, using GSKs can be expressed as shown in Equation 20.

$$PTDF_{ik,A} = \sum_{n \in A} GSK_n \cdot PTDF_{ik,n} \tag{20}$$

Where

$$\sum_{n \in A} GSK_n = 1 \tag{21}$$

One should have in mind that inaccurate GSKs may influence the market extensively, and may be one of the major sources of inaccuracies in flow-based (FB) market clearing (FBMC) [8].

The PTDFs can be used to calculate the flow of any given line in a system, P_{ik}, based on the NP of all nodes according to Equation 22, providing a methodology for power flow calculations.

$$P_{ik} = \sum_{n \in N} PTDF_{ik,n} \cdot NP_n \tag{22}$$

The transmission grid is the backbone of today's power system, and it is of great importance to develop decision making tools that are suited for future market environments outlining the cost recovery of grid investments, and to improve the power system modelling within these tools [12].

3.3 NTC and FB in context of an optimization program

The objective function of the optimization problem remains unchanged with the two aforementioned methodologies, NTC and FB. The only difference is the formulation of power flow constraints and possible inclusion of variables for voltage angels. Because there is no need for pre-allocation of capacity in advance of the FB market clearing, a larger solution domain can be obtained by the algorithm, still containing all possible solutions of the CNTC [8]. This implies that FB market clearing might contain solutions outside the solution domain of CNTC, providing a greater number of trading opportunities with the same level of security of supply [7]. This is illustrated by Figure 2.

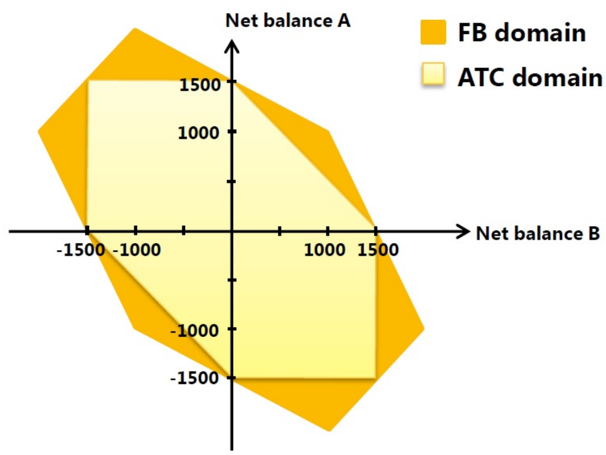

Figure 2: Illustration of NTC (ATC) compared with FB solution domain [7].

3.3.1 Expansion planning

There are numerous ways to model an expansion planning problem. One approach is the generic mixed-integer linear program (MILP) model described by Trötscher and Korpås in [13], NetOp. In the original version a transportation model of the grid was used, simply modelling branches as transmission capacity constraints, expressed as the NTC described in Section 3.1, neglecting the physical nature of electric power flows. However, due to the previously discussed limitations of NTC, PTDFs deducted in Section 3.2.2, can be utilized to model the interconnected power flows of the entire grid. The NTC only restrict flow on each connection, while the PTDFs are used to translate market transactions into physical power flows in the system, creating a stronger coupling between the power market and the physical system. This method provides a better, more realistic description of the grid than using a transportation model, while still maintaining linearity [13].

To account for the flow-based capacity restrictions and the use of PTDF, the optimization model has to be augmented with the additional constraint given in Equation 23. It takes into account all power flows in the interconnected system resulting from the net positions of all nodes. The PTDF-matrix can be used both statically and dynamically, implying that the latter is iteratively updated whenever the investment model adds additional transmission capacity.

$$\sum_{n \in I} PTDF^{s}_{ij,n} \left(\sum_{g \in G_n} x^{s}_{g} - b^{s}_{n} \right) \leq x^{s}_{ij}, \forall i \in I, j \in I_i, s \in S \tag{23}$$

4 Stochastic Programming

To give the reader some intuition behind how to incorporate uncertainty we will briefly introduce the basics of stochastic programming in this section. The information provided is based on [14]. It is strongly recommended to read the paper by Julia Higle [15], which given a good introduction to the field.

4.1 Introduction

The most common form is called two-stage stochastic linear programs with recourse, which means that you can "correct" your decisions made in the first stage with corrective actions in the second stage, i.e. after you gain more information about the uncertain parameters.

However, in many applications such as power system planning it is natural to consider multiple stages where you learn about new information. E.g. in expansion planning you might have several decision stages and a valuable property that makes the resulting problem block separable recourse which is something that can be utilized to solve the problem faster. This is where you would apply so called decomposition techniques. Heuristics are also applied to large problems in order to make them tractable.

4.1.1 Probability Space and Random Variables

Uncertainty is represented with random outcomes denoted by ω (scenarios). The set of all possible outcomes is usually denoted Ω. Some outcomes may be combined into subsets of Ω, usually referred to as *events*.

For a random variable ξ with the possible outcomes ω, there is a cumulative distribution $F_\xi(x) = P(\xi \leq x) = P(\omega | \xi \leq x)$.

A discrete variable takes a finite number of different values. It is best described by its probability distribution, i.e. a list of all possible ξ^k for $k \in K$ with probabilities

$$\begin{aligned} & f(\xi^k) = P(\xi = \xi^k) \\ & \sum_{k \in K} f(\xi^k) = 1 \end{aligned} \tag{24}$$

While dicrete random variables are described with probability distribution, continous random variables are described through desity functions $f(\xi)$. The probability of ξ having an outcome in the interval [a,b] is obtained as

$$\begin{aligned} P(a \leq \xi \leq b) &= \int_a^b f(\xi) d\xi \\ P(a \leq \xi \leq b) &= \int_a^b dF(\xi) \end{aligned} \tag{25}$$

4.1.2 Decisions Stages

Stochastic linear programs are linear programs where some data might be considered uncertain. First stage variables are usually denoted as x, while second stage variables often are denoted $y(\omega, x)$ to stress the fact that they differ as functions of the outcome and the first stage decision. We get the following sequence of events and decisions:

$$x \to \xi(\omega) \to y(\omega, x) \tag{26}$$

4.1.3 Two-Stage Program with Fixed Recourse

The classical formulation is:

$$\begin{aligned} &\min Z = c^T x + E_\xi[\min q(\omega)^T y(\omega)] \\ &\text{s.t.} \\ &Ax = b \\ &T(\omega)x + Wy(\omega) = h(\omega) \\ &x \geq 0, y(\omega) \geq 0 \end{aligned}$$

c, b, and A are associated with the first stage variables. In the second stage the dependent parameters are dependent on the realization of $\omega \in \Omega$, i.e. the parameters are known after the new information is learned for $q(\omega)$, $h(\omega)$, and $T(\omega)$.

The objective function Z contains both a deterministic terms and a expectation of the second stage over all realizations of ω. In the second stage $y(\omega)$ is a solution of a linear problem, which makes this a bit complicated. To stress this, one often uses to formulate as deterministic equivalent, where the second stage value function (a optimization problem itself)

$$Q(x, \xi(\omega)) = \min_y [q(\omega)^T y | Wy = h(\omega) - T(\omega)x, y \geq 0] \tag{27}$$

can be used to derive the expected second stage value function

$$L(x) = E_\xi Q(x, \xi(\omega)) \tag{28}$$

together providing the DEP

$$\begin{aligned} &minZ = c^T x + L(x) \\ &\text{s.t.} \\ &Ax = b \\ &x \geq 0 \end{aligned} \tag{29}$$

4.1.4 Random Variables and Risk Aversion

One can think of two different classes for random events and random variables, i.e. a) uncertainties that occur frequently on short-term basis (e.g. daily or weekly demand), and b), uncertainties that represents some scenarios, where only a small number is realized in the long-term.

Let's look at an Airline company that makes thousands of schedules a month, in such case one could expect that one would receive around the expected revenue each month (maybe also day). Risk aversion has no, or little, effect in such a case (a).

If one looks at case b), where an objective function that only considers the expected value of the second stage, might give a poor representation of risk aversion.

In practical applications, risk aversion is often represented with a piecewise-linear utility function (which in reality are non-linear). One can also include a linear constraint, called *downside risk*. The expected downside risk is simply calculated over all scenarios, and the resulting constraint for this means that the expected downside risk must fall below some level. An example:

$$\begin{aligned}
&max \quad Z = c^T x + E_\xi[max(q(\omega)^T y(\omega)] \\
&\text{s.t.} \\
&Ax = b \\
&T(\omega)x + Wy(\omega) = h(\omega) \\
&x \geq 0, y(\omega) \geq 0
\end{aligned}$$

where we want some target level g for the profit Z. The downside risk $u(\xi)$ is thus defined by two constraints:

$$\begin{aligned}
&u(\xi(\omega)) \geq g - q^T(\omega)y(\omega) \\
&u(\xi(\omega)) \geq 0
\end{aligned}$$

and the expected risk

$$E_\xi u(\xi) \leq l \tag{30}$$

where l is some given level. The constraint is linear for discrete random vector ξ. It is also a first stage constraints, since it runs over all scenarios, i.e. it can be included in the extensive form.

4.1.5 Probabilistic Programming

If costs and benefits in the second stage is difficult to assess, then probabilistic programming may be useful where some constraint or objective function are expressed in terms of probabilistic statements about the fist stage decisions.

Deterministic linear equivalent (direct case): An airline company that want to portion seats in to different classes, i.e. 98 percent probability to cover business class and 95 percent chance to cover economy demand

Deterministic linear equivalent (indirect case)

Deterministic nonlinear equivalent (the case of random constraint coefficients): The dieet problem where you want to select a number of foods in order to get the cheapest menus that meet nutrients requirements (daily)

4.1.6 Risk Measures

Risk aversion can as mentioned be modelled as pice-wise linear utility functions. In many applications, it is usefull to express it through *Value at Risk (VaR)*. However, the weakness with VaR as a risk measure is that it does not hold for *subadditivity*, which is one of the requisits for *coherent risk measures*:

1 Subadditivity	$R(\xi+\zeta) \leq R(\xi) + R(\zeta)$
2 Positive Homogeneity	$R(\alpha\xi) = \alpha R(\xi) \forall \alpha \geq 0$
3 Monotonicity	$P(\xi) \leq P(\zeta)$ whenever $\xi \preceq \zeta$
4 Translation Invariance	$R(\xi + t) = R(\xi) + t$ for any $t \in \Re$

where $\preceq$ means first-order stochastic dominance; $R(\xi \leq t) \geq R(\zeta \leq t)$ for all t

To satisfy these risk measures one can use a related risk measure *conditional value at risk (CVaR)* which takes the conditional expectations over losses that excess *VaR*. For a random loss ξ with distribution function P, the α-confidence level is defined

$$CVaR_\alpha(\xi) = E_{P_\alpha}[\xi] \tag{31}$$

where P_α is the distribution function.

4.2 Value of Information and the Stochastic Solution

Since SP in general is harder to model and solve it is of interest to measure the potential trade-off between deterministic og stochastic programs. To answer this we study two consepts, a) the Expected Value of Perfect Information (EVPI), and b), the Value of the Stochastic Solution (VSS).

4.2.1 EVPI

EVPI measures the maximum amount a decion maker would be ready to pay in return for complete information about the future. For example, let us model uncertainty with a finite amount of scenarios where ξ is the random variable whose realizations corresponds to the various scenarios, we get the following optimization problem for one particular scenario ξ

$$
\begin{aligned}
&min \quad Z(x,\xi) = c^T x + min[q^T y \quad | \quad Wy = h - Tx, \quad y \geq 0] \\
&\text{s.t.} \quad Ax = b \quad , x \geq 0
\end{aligned}
$$

One could search for optimal soultions thorugh all scenarios, which is know as the *distribution problem*, i.e. a generalization of sensitivity analysis or parametric analysis or parametric analysis in LP. The expected value of all these solutions is known as the *wait and see* solution:

$$WS = E_\xi[\min_x Z(x,\xi)] = E_\xi[Z(\bar{x}(\xi),\xi)] \tag{32}$$

We want to compare the WS with the so called *here and now* solution, which correspons to the recourse problem (RP):

$$RP = \min_x \quad E_\xi Z(x,\xi) \tag{33}$$

with an optimal soultion x^*. The EVPI is the difference between the RP and the WS soultion:

$$EVPI = RP - WS \tag{34}$$

4.2.2 VSS

It is not easy to always measure the WS solution due to lack of information, and that it delivers a set of soultions instead of one solution. It might therefore be more tempting to solve the *expected value problem* or *mean value problem* where all random variables are replaced by their expected values.

$$EV = \min_x \quad Z(x,\bar{\xi}) \tag{35}$$

where $\bar{\xi} = E(\xi)$ is the expectation of ξ. The VSS measures how good, or more frequently, how bad a decision $\bar{x}(\bar{\xi})$ (optimal soultion of EV) is in terms of RP. However, let us first define the *expected result of using the EV soultion*:

$$EEV = E_\xi[Z(\bar{x}(\bar{\xi}),\xi)] \tag{36}$$

EEV measures how the expected value soultion, $\bar{x}(\bar{\xi})$, performs when allowing the second stage decision to be chosen optimally as functions of $\bar{x}(\bar{\xi})$ and ξ. The VSS becomes

$$VSS = EEV - RP \tag{37}$$

Which is the cost of ignoring uncertainty in decision making.

4.2.3 Relation between EVPI and VSS

For any stochastic program

$$
\begin{aligned}
EVPI &\geq 0 \\
VSS &\geq 0
\end{aligned}
\tag{38}
$$

For stochastic programs with *fixed recourse matrix* and *fixed objective coefficients*

$$
\begin{aligned}
EVPI &\leq EEV - EV \\
VSS &\leq EEV - EV
\end{aligned}
\tag{39}
$$

4.3 Solution Methods

Many SP has a structure that can be utilized to solve the problems faster and more efficiently. The block structure that are found in two-stage problems can also be seen in multistage problems, hence we can also use the same decomposition techniques.

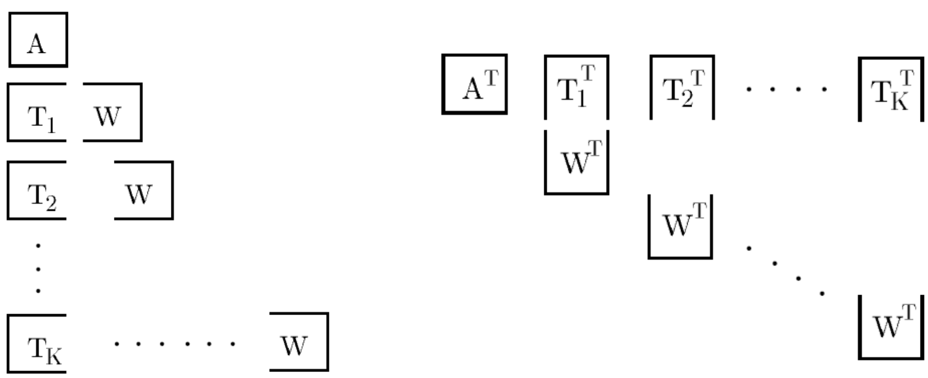

Figure 3: Primal (right)- and dual (left) blockstructure of a Two-stage program [14]

The most common techniques are called *cutting plane techniques* which is based on an outer linearization of the cost-to-go-function, i.e. the second stage costs. In this section we will explore the cutting method, proof of convergence, and enhancements in terms of multicuts and branching of realizations.

Other variants consider use of bounding techniques, and combinations with sampling methods.

Alternative decomposition procedures spans from *inner linearization* (Dantzig-Wolfe) that solved the dual of the L-shaped method problem, to a *primal form of inner linearization* based on generalized programming. Also direct approaches such as *efficient extreme points* and *interior point* methods are discussed.

5 Mathematical model formulation

This section presents the mathematical formulation of the expansion planning model (PowerGIM). Consult the paper on NetOp [16] for a deterministic formulation of a MILP for grid investments. This is more or less the same model presented in this section but with adaptions to the Python environment used in the LP market simulator PowerGAMA [17] and [18]. In addition, the model is extended to incorporate uncertainty and CO2 emissions, as presented in e.g. [19]. A nomenclature is included in the appendix.

The model assumes perfect competition, inelastic demand and a centralized planner for multinational grid investments. It is targeted for system characteristics in the North Sea region where both offshore grid technology costs and hydro representation plays an important role. The resulting problem is an open-loop equilibrium model with decisions regarding transmission capacity in the upper-level, and generators' response by capacity investments and dispatch in the lower levels. This allows generators to react upon transmission investments, as the grid investments have a considerable material impact on expected market prices. By assuming perfect competition and inelastic demand, such an equilibrium model can be reformulated into an optimization problem (Samuelson, 1952) and (Munoz, 2016).

5.1 Compact formulation

A compact formulation of the stochastic MILP is given in this section. In order to incorporate uncertainty, the model is formulated as a two-stage stochastic program which relates to a mixed-integer linear program (MILP) in its extensive form. Integer variables are used to decide upon transmission infrastructure investments in the first stage, while the second stage problem is a pure linear program (LP) reflecting generator capacity investment and market operation. By only considering one scenario, the model is equivalent to a deterministic program. A compact model formulation of the stochastic MILP is given in (40) below.

$$TC = \min_{x} c^T x + E_\xi[\min_{y(\omega)} q(\omega)^T y(\omega)] \tag{40a}$$

s.t.

$$Ax \leq b \tag{40b}$$

$$T(\omega)x + Wy(\omega) \leq h(\omega), \quad \forall \omega \in \Omega \tag{40c}$$

$$x = (x_1, x_2) \geq 0, x_1 \in \{0,1\}, x_2 \in Z^+, y(\omega) = (y_1(\omega), y_2(\omega)) \geq 0, \quad \forall \omega \in \Omega$$

In (40a), the objective function is divided into two stages; first the costs related to infrastructure investments, and second, the expected costs of market operation, $y_1(\omega)$, and generator capacity investments, $y_2(\omega)$, dependent on a discrete set of scenarios, Ω. For the work presented in this paper, generator capacity investments are disregarded in order to narrow down the scope to grid investments.

The vectors and matrices c, b, and A are associated with the first stage variables, i.e. investment in grid infrastructure. The cost vector c comprise of both fixed and variable node- and branch costs. Vector b restricts the investment decisions, e.g. by the maximum allowed capacity per investment block (e.g. 1000 MW per branch), and A is the corresponding coefficient matrix to those investment constraints.

The second stage parameters are dependent on the realization of $\omega \in \Omega$, i.e. the parameters are not quantified before uncertainty is revealed. The cost vector $q(\omega)$ is the marginal cost of generation and the capital capacity costs for generation. The right-hand-side vector $h(\omega)$ is the restrictions for scenario ω, i.e. relevant restrictions on market dispatch and investments in generator capacity. The transition matrix $T(\omega)$ is associated with first stage investments and it contains scenario and/or time-dependent data affecting operation in the second stage. The recourse matrix, W, is considered fixed in this model since as the coefficients in the matrix are independent of the realization of ω.

We could also formulate the second stage separately as a pure LP, to take advantage of the deterministic equivalent formulation;

$$Q(x,(\Omega,\pi)) = \min_{y(\omega)}[q(\omega)^T y(\omega) \quad | \quad Wy(\omega) = h(\omega) - T(\omega)x \quad , y(\omega) \geq 0] \tag{41}$$

together allowing us to write the deterministic equivalent as

$$\begin{aligned} & TC((\Omega,\pi)) = \min_x c^T x + E_\omega[Q(x,(\Omega,\pi))] \\ & \text{s.t.} \\ & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{42}$$

5.2 Extensive formulation

In this formulation, we have chosen to represent uncertainty in generation capacity and demand, unlike most other stochastic TEP models, that uses time step or hour as scenarios. Hence, each capacity scenario is operated over N time steps given by the sampling or clustering algorithm, which also is decoupled in time due to no time-dependency in the modelled system (storage, ramping etc.). Given four capacity scenarios and N=50 samples is equivalent to an hourly-scenario model with 200 scenarios, in a two-stage setting.

$$\min_{x,y} \quad FSC + a\frac{8760}{|T|}\sum_{s\in S}\pi_s SSC_s \tag{2a}$$

$$\text{s.t.} \qquad FSC = \sum_{j\in B}(C_j^{fix}y_j + C_j^{var}x_j) + \sum_{n\in N} C^{node}y_n \tag{2b}$$

$$SSC_s = \sum_{i\in G}\sum_{t\in T}(MC_i + CO2_i)x_{its} + \sum_{n\in N}\sum_{t\in T} VOLL x_{nts} + \sum_{i\in G} CX_i x_i \quad \forall s \in S \tag{2c}$$

$$\sum_{i\in G_n} x_{its} + \sum_{j\in B_n^{in}} x_{jts}(1-l_j) - \sum_{j\in B_n^{out}} x_{jts} + x_{nts} = \sum_{l\in L_n} D_{lts} \quad \forall n \in N, t\in T, s\in S \tag{2d}$$

$$x_{nts} \leq \sum_{l\in L_n} D_{lts} \quad \forall n \in N, t\in T, s\in S \tag{2e}$$

$$C_j^{fix} = B + B^d D_j + 2CL/CS \quad \forall j \in B \tag{2f}$$

$$C_j^{var} = B^{dp} D_j + 2CL^p/CS^p \quad \forall j \in B \tag{2g}$$

$$P_{it}^{min} \leq x_{its} \leq P_{it}^{max} + x_i \quad \forall i \in G, t\in T, s\in S \tag{2h}$$

$$-(P_j^e + x_j) \leq x_{jts} \leq (P_j^e + x_j) \quad \forall j \in B, t\in T, s\in S \tag{2i}$$

$$x_j \leq P_j^{n,max} y_j \quad \forall j \in B \tag{2j}$$

$$\sum_{j\in B_n} y_j \leq M y_n \quad \forall n \in N \tag{2k}$$

$$x_j, x_{its}, x_{jts}, x_{nts} \geq 0, \quad y_j \in Z^+, \quad y_n \in \{0,1\}$$

The objective function (2a) sums over the first stage costs (2b) and second stage costs (2c), which represents investment- and expected operational costs, respectively. The expected operational costs are scaled up to represent one year with respect to the a weighting factor of each sample considered, and then multiplied with an annuity factor to get the net present value over L years of operation and r interest rate.

Kirchhoff's current law (KCL), i.e. the nodal energy balance, is represented in (2d) which ensures that generated power and power flows at connected branches at each node, is in balance with the nodal demand. A load shedding variable is added to the equation in order to allow for load shedding, but at a cost of VOLL in the objective function.

Equation (2i), (2j) and (2k) describes the branch flow (transport model) and investment limits in new transmission capacity. See documentation of PowerGAMA for the DC load flow equations, which easily can be extended to PowerGIM but at the expense of using binary variables to describe DC load flow equations for new branches.

Non-anticipative constraints are foregone due to the fact that each decision variable is stage dependent and that the first stage variables that only are exposed to an uncertain future in the first node. However, for multistage extensions it is necessary to include non-anticipativity constraints in order to force stage-dependent decision variables to be the same for scenario branches through a given stage node.

5.3 Solution procedure

A deterministic equivalent is simply solved as extensive form that is passed to a solver, e.g. Gurobi or Cplex. However, when you add scenarios to the problem, which is the case for the stochastic program, the complexity and size increase significantly. In some cases, your computer might be able to handle an extensive form of the problem, but usually the user needs to apply decomposition techniques and heuristics to get a solution.

5.3.1 Progressive Hedging Algorithm (PHA)

The progressive hedging algorithm (PHA) is based on scenario-wise decomposition and very similar to the Lagrangian relaxation, in terms of relaxing the non-anticipative constraints and adding penalty terms to the objective function. Nodes in the decomposed scenario tree that are supposed to represent their original node in the scenario tree, are penalised in the objective function until they provide the same decision variables. Intuitively speaking, you have to make the same decision in nodes where you have the same knowledge about the future.

6 Model Structure

PowerGIM uses about the same format as PowerGAMA. This section will give a brief introduction to the model structure.

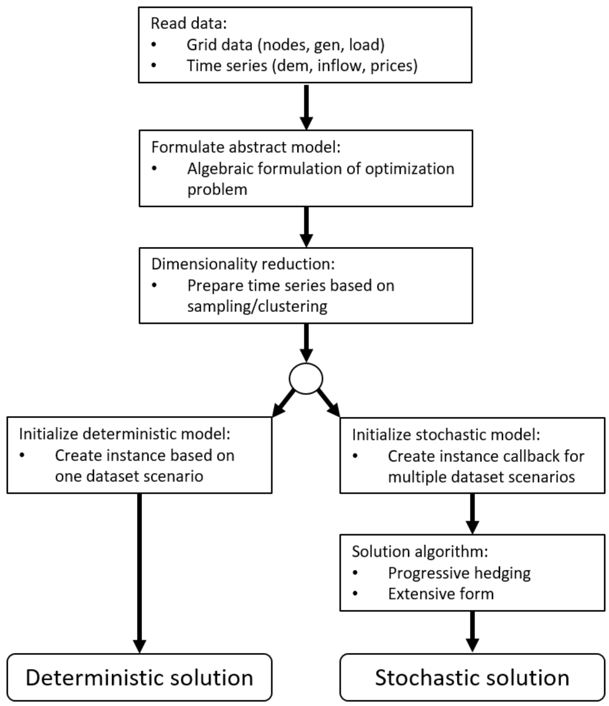

Figure 4: PowerGIM task flow chart

6.1 Input data format

Data input from .csv files. Tables 1, 2, 3, 4 describe the format of the CSV input data files for nodes, branches, generators and consumers respectively.

6.1.1 Network model data (CSV files)

Table 1: CSV Input data: Nodes

column key	description	type	units
"id"	Unique string identifier	string	
"lat"	Latitude	float	degrees
"lon"	Longitude	float	degrees
"area"	Area/country code	string	
"existing"	Whether node already exists (0/1)	int	
"offshore"	Whether node is offshore (0/1)	int	
"cost_scaling"	Scaling factor for costs	float	
"type"	Node type (e.g. AC or DC)	string	

Table 2: CSV Input data: Branches

column key	description	type	units
"node_from"	Node identifier	string	
"node_to"	Node identifier	string	
"capacity"	Capacity	float	MW
"capacity2"	(Optional) Additional capacity before stage 2	float	MW
"expand"	Which stage to consider expansion (1,2,0=not)	int	
"cost_scaling"	Scaling factor for costs	float	
"type"	Branch type	string	
"distance"	(optional) Branch distances (computed if not given)	float	km

Table 3: CSV Input data: Generators

column key	description	type	units
"node"	Node identifier	string	
"desc"	(Optional) Description or name	string	
"type"	Generator type	string	
"pmax"	Capacity (maximum production)	float	MW
"pmax2"	(Optional) Additional capacity before stage 2	float	MW
"pmin"	Minimum production	float	MW
"fuelcost"	Cost of generation	float	/MWh
"fuelcost_ref"	Reference to time-series	string	
"pavg"	(Optional) Limit on average production (0=no limit)	float	MW
"inflow_fac"	Inflow factor	float	MW
"inflow_ref"	Inflow profile reference	string	
"expand"	Which stage to consider expansion (1,2,0=not)	int	
"cost_scaling"	Scaling factor for costs	float	
"p_maxNew"	(Optional) Maximum new capacity	float	MW

Table 4: CSV Input data: Consumers

column key	description	type	units
"node"	Node identifier	string	
"demand_avg"	Average demand	float	MW
"demand_ref"	Reference to time-series	string	
"emission_cap"	(Optional) Maximum CO2 emmission allowed	float	X

6.1.2 Parameters (XML file)

Tables 5, 6, 7, 8 describe the content of the XML input file containing information about node types, branch types, generator types and other parameters respectively.

Table 5: XML Input data: Node type items

attribute	description	type	units
name	Node type identifier	string	
L	Cost for node on land	float	EUR/MW
S	Cost for node at sea (offshore)	float	EUR/MW

Table 6: XML Input data: Branch type items

attribute	description	type	units
name	Branch type identifier	string	
Bdp	Cost per distance and power	float	EUR/MW/km
Bd	Cost per distance	float	EUR/MW
B	Fixed cost per cable set	float	EUR
CLp	Cost per power per on-land endpoint	float	EUR/MW
CL	Fixed cost per on-land endpoint	float	EUR
CSp	Cost per power per offshore endpoint	float	EUR/MW
CS	Fixed cost per offshore endpoint	float	EUR
lossFix	Loss factor, fixed part	float	
lossSlope	Loss factor, distance dependent part	float	/km

Table 7: XML Input data: Generator type items

attribute	description	type	units
name	Node type identifier	string	
CX	Investment cost	float	EUR/MW
CO2	CO2 emission rate	float	kg/MWh

Table 8: XML Input data: Parameters

attribute	description	type	units
financeInterestrate	Interest rate for net present value calculations	float	
financeYears	Lifetime for cost-benefit calculations	int	
omRate	Operation and maintenance costs relative to investment	float	
curtailmentCost	Penalty for curtailment of renewable energy	float	EUR/MWh
CO2price	Price for CO2 emmissions	float	EUR/kg
VOLL			
stage2TimeDelta	Time delay between stage 1 and stage 2 investments	int	years

6.2 Time series

Time series are given in relative terms from 0 to 1 for non-dispatchable power generation and load profiles. One could also use time series to reflect variations in marginal costs.

6.3 Illustrations

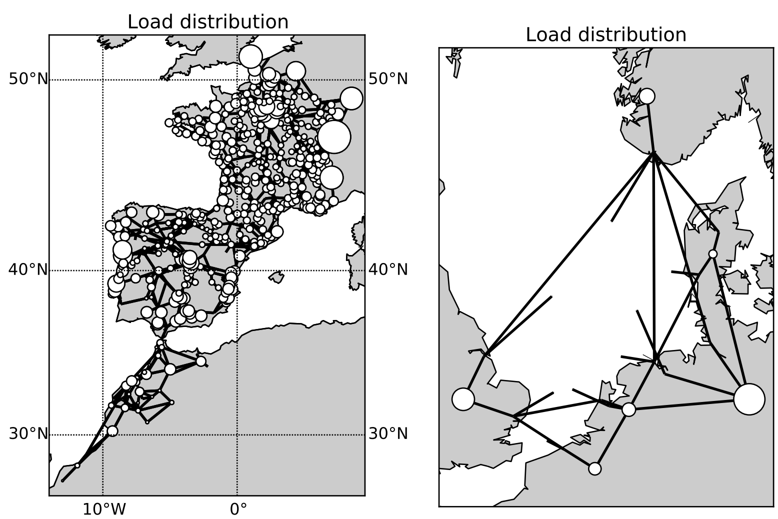

Figure 5: Load distribution in SW Europe and Nordic

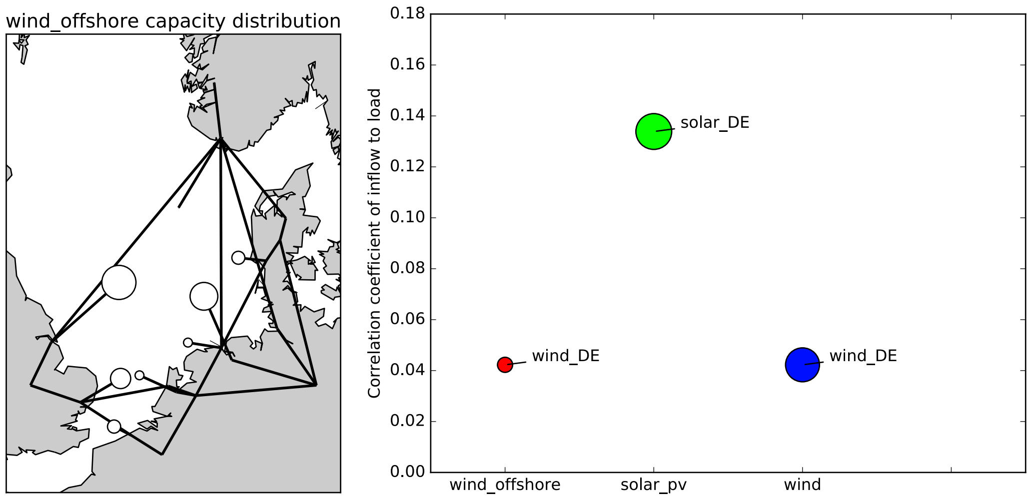

Figure 6: Generation distribution by technology (offshore wind) and scatter plot of VRES correlation w/load

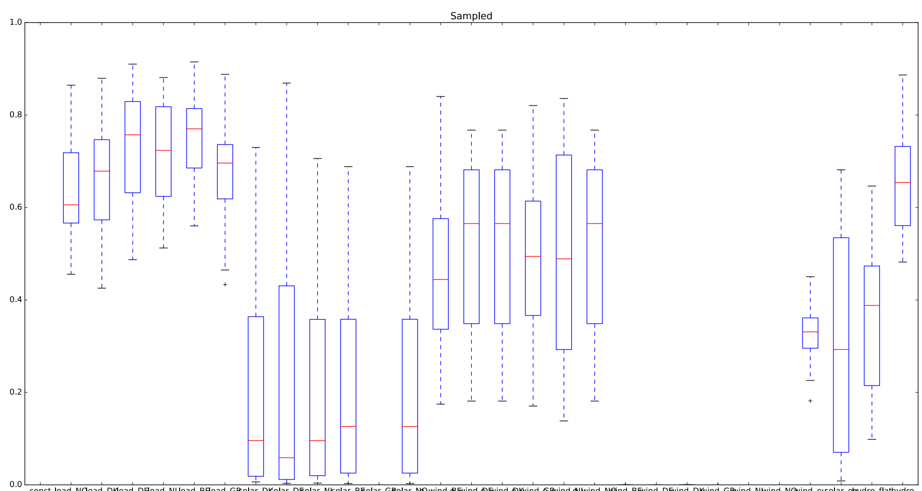

Figure 7: Box plot of sampled (N=50 Kmeans++) time series

7 Result examples

The optimization model has been tested on data sets for the North Sea area with 2030 scenarios. The data sets are in the same format as in NetOp and does only consider transmission expansion (not generation). First stage variables are the investment decisions to be made on transmission capacity, and the second stage represents the market operation for cost recovery calculations for the investment expenses. There are four different second stage scenarios, each representing the four distinctive ENTSO-E visions for 2030.

The optimal solution from the SP model can deviate quite significantly from different evaluations a of classical deterministic scenario analysis. One could also compare with the expected value scenario, and metrics concerning the value of stochastic solution (VSS), expected value of perfect information (EVPI) and value added by flexibility (when considering multiple investment stages).

At the current stage, the model makes investment decisions under static circumstances, meaning that the investment decisions are all made the same year. Construction time, lead time and multiple investment possibilities are currently ignored.

7.1 Expansion Plot

A plot of capacity expansion. The thickness/color of the line represents the capacity investment.

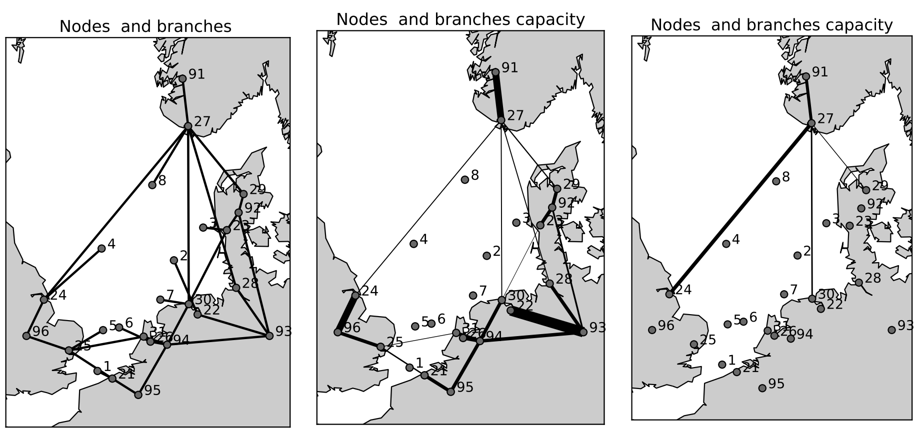

Figure 8: From left 1. candidate lines, 2. initial capacities, and 3. new capacities

7.2 Branch Utilization

A histogram of branch flows and a box-plot or violin-plot of branch flows.

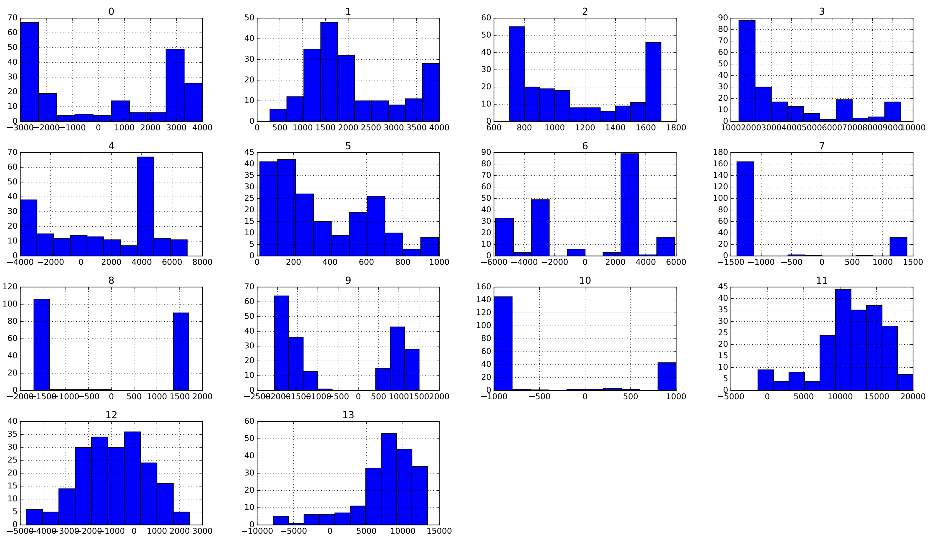

Figure 9: Branch flow histogram

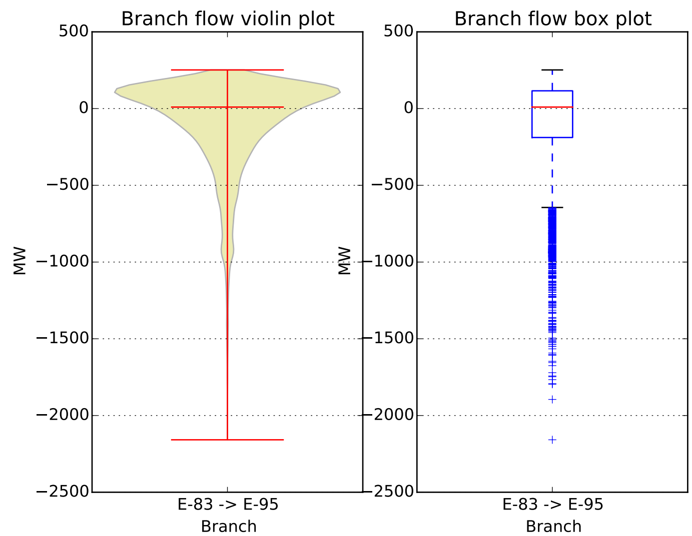

Figure 10: Branch flow histogram

For more information regarding documentation and applications of PowerGAMA, please consult [20], [17] and [21].

Regarding the investment module, PowerGIM, it is recommended to study [16] for the basic concepts around this model and it's current application in the North Sea area.

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8 Appendix

Sets & Mappings		
	$n \in N$	: nodes
	$i \in G$	: generators
	$j \in B$	: branches
	$l \in L$	: loads, demand, consumers
	$t \in T$	: time steps, hour
	$s \in S$	: scenarios
	$i \in G_n, l \in L_n$	: generators/load at node n
	$n \in B_n^{in}, B_n^{out}$	: branch in/out at node n
	$n(i), n(l)$	: node mapping to generator i/load unit l
Parameters		
	r	: interest rate
	r^{om}	: operations and maintenance rate
	L	: economic lifetime
	a	: annuity factor
	π_s	: probability for scenario s
	$VOLL$	: value of lost load (cost of load shedding)
	MC_i	: marginal cost of generation (EUR/MWh)
	$CO2_i$	: co2 price for generator i (EUR/MWh)
	D_{lts}	: demand at load l, hour t, scenario s
	B, B^d, B^{dp}	: branch costs (mobilisation, fixed- and variable cable cost)
	CL, CL^p	: onshore switchgear (fixed and variable cost)
	CS, CS^p	: offshore switchgear (fixed and variable cost)
	CX_i	: capacity cost for generator g (EUR/MW)
	NL, NS	: onshore/offshore node costs (e.g. platform costs)
	P_{its}^{min}	: minimum generation capacity, generator i, hour t, scenario s
	P_{its}^{max}	: maximum generation capacity, generator i, hour t, scenario s
	P_j^e	: existing branch capacity, branch j
	$P_j^{n,max}$	: maximum new branch capacity, branch j
	D_j	: distance/length, branch j
	l_j	: transmission losses (fixed + variable wrt distance), branch j
	E_i	: yearly disposable energy for generator i (e.g. energy storage)
	M	: a sufficiently large number
	B_{nk}, H_{jk}	: line/node susceptance/network transfer matrices
Primal variables		
	y_j	: number of new transmission lines/cables, branch j
	y_n	: new platform/station, node n
	x_j	: new transmission capacity, branch j
	x_i	: new generation capacity, generator i
	x_{its}	: power generation dispatch, scenario s, generator i, hour t
	x_{jts}	: power flow, branch j, hour t, scenario s
	x_{nts}	: load shedding, node n, hour t, scenario s
	δ_{nts}	: voltage angle, node n, hour t, scenario s

Table 9: Notation for the two-stage stochastic transmission expansion planning model