CCZ4gauge+MHD+tabEoS+leakage+LES

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Fields

Sfd_x, Sfd_y, Sfd_z, Bfu_x, Bfu_y, Bfu_z, Df, DYf, $tauf$, $phif$, gtd_xx, gtd_xy, gtd_xz, gtd_yy, gtd_yz, gtd_zz, Atd_xx, Atd_xy, Atd_xz, Atd_yy, Atd_yz, Atd_zz, $Gamh\_x$, $Gamh\_y$, $Gamh\_z$, $Betau\_x$, $Betau\_y$, $Betau\_z$, $Alpha$, chi, trK, $theta$

Spatial Coordinates

x, y, z

Time Coordinate

t

Auxiliary Fields

ρf, Yef, vfd_x, vfd_y, vfd_z, pf, $epsf$, Tf, $sqcs$, qnu, rnu, $optdepthe$, $optdeptha$, $optdepthx$, $chie$, $chia$, $chix$, $t\_optdepthe$, $t\_optdeptha$, $t\_optdepthx$, t_chie, t_chia, t_chix, Efu_x, Efu_y, Efu_z, $TauN\_x$, $TauN\_y$, $TauN\_z$, $TauNe\_x$, $TauNe\_y$, $TauNe\_z$, $TauM\_xy$, $TauM\_xz$, $TauM\_yz$, $TauT\_xx$, $TauT\_xy$, $TauT\_xz$, $TauT\_yy$, $TauT\_yz$, $TauT\_zz$, $dpfdeps$, $dpfdrho$, $dpfdye$, qnu_a, qnu_e, qnu_x

Auxiliary Variables

Bvf, $optdepthеh$, $optdepthah$, $optdepthxh$, $optdepthev$, $optdepthav$, $optdepthxv$, $optdepthed$, $optdepthad$, $optdepthxd$, $chieh$, $chiah$, $chixh$, $chiev$, $chiav$, $chixv$, $chied$, $chiad$, $chixd$, sqa, $sqca$, $sqbc$, h, D, tau, Sd_x, Sd_y, Sd_z, Su_x, Su_y, Su_z, Bd_x, Bd_y, Bd_z, Bu_x, Bu_y, Bu_z, W, $faceta$, $kappa\_cc$, $kappa\_z1$, $kappa\_z2$, $feta$, $kappa\_f$, chi_max, inv_chi, $detgtd$, $idetgtd$, gtu_xx, gtu_xy, gtu_xz, gtu_yy, gtu_yz, gtu_zz, $sdetg$, vfu_x, vfu_y, vfu_z, Bfd_x, Bfd_y, Bfd_z, Sfu_x, Sfu_y, Sfu_z, Efd_x, Efd_y, Efd_z, $Thetaf$, $sqvf$, $sqBf$, $Bfvf$, $sqEf$, sqW, $invsqW$, sqW, $Sfuu\_xx$, $Sfuu\_xy$, $Sfuu\_xz$, $Sfuu\_yy$, $Sfuu\_yz$, $Sfuu\_zz$, $Tf4u\_tt$, $Tf4u\_tx$, $Tf4u\_ty$, $Tf4u\_tz$, $Tf4u\_xx$, $Tf4u\_xy$, $Tf4u\_xz$, $Tf4u\_yy$, $Tf4u\_yz$, $Tf4u\_zz$, Tu_tt, Tu_tx, Tu_ty, Tu_tz, Tu_xx, Tu_xy, Tu_xz, Tu_yy, Tu_yz, Tu_zz, rho_ADM, Jtd_ADM_x, Jtd_ADM_y, Jtd_ADM_z, $Betatd\_x$, $Betatd\_y$, $Betatd\_z$, $pTtd\_ADM\_xx$, $pTtd\_ADM\_xy$, $pTtd\_ADM\_xz$, $pTtd\_ADM\_yy$, $pTtd\_ADM\_yz$, $pTtd\_ADM\_zz$, tr_pT, $Atud\_xx$, $Atud\_xy$, $Atud\_xz$, $Atud\_yx$, $Atud\_yy$, $Atud\_yz$, $Atud\_zx$, $Atud\_zy$, $Atud\_zz$, $trAt$, Atu_xx, Atu_xy, Atu_xz, Atu_yy, Atu_yz, Atu_zz, Ctd_xxx, Ctd_xxy, Ctd_xxz, Ctd_xyy, Ctd_xyz, Ctd_xzz, Ctd_yxx, Ctd_yxy, Ctd_yxz, Ctd_yyy, Ctd_yyz, Ctd_yzz, Ctd_zxx, Ctd_zxy, Ctd_zxz, Ctd_zyy, Ctd_zyz, Ctd_zzz, Ct_xxx, Ct_xxy, Ct_xxz, Ct_xyy, Ct_xyz, Ct_xzz, Ct_yxx, Ct_yxy, Ct_yxz, Ct_yyy, Ct_yyz, Ct_yzz, Ct_zxx, Ct_zxy, Ct_zxz, Ct_zyy, Ct_zyz, Ct_zzz, div_Beta, $d\_div\_Beta\_x$, $d\_div\_Beta\_y$, $d\_div\_Beta\_z$, $Gamt\_x$, $Gamt\_y$, $Gamt\_z$, Zu_x, Zu_y, Zu_z, Rpd_xx, Rpd_xy, Rpd_xz, Rpd_yy, Rpd_yz, Rpd_zz, Rtd_xx, Rtd_xy, Rtd_xz, Rtd_yy, Rtd_yz, Rtd_zz, $Rscalar$, Psi_1_xx, Psi_1_xy, Psi_1_xz, Psi_1_yy, Psi_1_yz, Psi_1_zz, $trPsi_1$, Psi_1TF_xx, Psi_1TF_xy, Psi_1TF_xz, Psi_1TF_yy, Psi_1TF_yz, Psi_1TF_zz, $Sfud\_xx$, $Sfud\_xy$, $Sfud\_xz$, $Sfud\_yx$, $Sfud\_yy$, $Sfud\_yz$, $Sfud\_zx$, $Sfud\_zy$, $Sfud\_zz$, $trSf$, $kappa\_f$, $decay\_factor$, Epf, $CovdinvsqW\_x$, $CovdinvsqW\_y$, $CovdinvsqW\_z$, $CovdDf\_x$, $CovdDf\_y$, $CovdDf\_z$, $CovdDYf\_x$, $CovdDYf\_y$, $CovdDYf\_z$, $CovdBvf\_x$, $CovdBvf\_y$, $CovdBvf\_z$, $Covdrhof\_x$, $Covdrhof\_y$,

$Covdrhof\_z$, $Covdpf\_x$, $Covdpf\_y$, $Covdpf\_z$, $Covdepsf\_x$, $Covdepsf\_y$, $Covdepsf\_z$, $Covdhf\_x$, $Covdhf\_y$, $Covdhf\_z$, $CovdEpf\_x$, $CovdEpf\_y$, $CovdEpf\_z$, $CovdThetaf\_x$, $CovdThetaf\_y$, $CovdThetaf\_z$, $Covddpfdeps\_x$, $Covddpfdeps\_y$, $Covddpfdeps\_z$, $Covddpfdrho\_x$, $Covddpfdrho\_y$, $Covddpfdrho\_z$, $Covddpfdye\_x$, $Covddpfdye\_y$, $Covddpfdye\_z$, $CovdYef\_x$, $CovdYef\_y$, $CovdYef\_z$, $CovuYef\_x$, $CovuYef\_y$, $CovuYef\_z$, $CovuinvsqW\_x$, $CovuinvsqW\_y$, $CovuinvsqW\_z$, $CovuBvf\_x$, $CovuBvf\_y$, $CovuBvf\_z$, $Covurhof\_x$, $Covurhof\_y$, $Covurhof\_z$, $Covuepsf\_x$, $Covuepsf\_y$, $Covuepsf\_z$, $Covuhf\_x$, $Covuhf\_y$, $Covuhf\_z$, $CovuThetaf\_x$, $CovuThetaf\_y$, $CovuThetaf\_z$, $Covdvfu\_xx$, $Covdvfu\_xy$, $Covdvfu\_xz$, $Covdvfu\_yx$, $Covdvfu\_yy$, $Covdvfu\_yz$, $Covdvfu\_zx$, $Covdvfu\_zy$, $Covdvfu\_zz$, $Covdvfd\_xx$, $Covdvfd\_xy$, $Covdvfd\_xz$, $Covdvfd\_yx$, $Covdvfd\_yy$, $Covdvfd\_yz$, $Covdvfd\_zx$, $Covdvfd\_zy$, $Covdvfd\_zz$, $Covuvfu\_xx$, $Covuvfu\_xy$, $Covuvfu\_xz$, $Covuvfu\_yx$, $Covuvfu\_yy$, $Covuvfu\_yz$, $Covuvfu\_zx$, $Covuvfu\_zy$, $Covuvfu\_zz$, $CovdBfu\_xx$, $CovdBfu\_xy$, $CovdBfu\_xz$, $CovdBfu\_yx$, $CovdBfu\_yy$, $CovdBfu\_yz$, $CovdBfu\_zx$, $CovdBfu\_zy$, $CovdBfu\_zz$, $CovdBfd\_xx$, $CovdBfd\_xy$, $CovdBfd\_xz$, $CovdBfd\_yx$, $CovdBfd\_yy$, $CovdBfd\_yz$, $CovdBfd\_zx$, $CovdBfd\_zy$, $CovdBfd\_zz$, $CovuBfu\_xx$, $CovuBfu\_xy$, $CovuBfu\_xz$, $CovuBfu\_yx$, $CovuBfu\_yy$, $CovuBfu\_yz$, $CovuBfu\_zx$, $CovuBfu\_zy$, $CovuBfu\_zz$, $CovdEfu\_xx$, $CovdEfu\_xy$, $CovdEfu\_xz$, $CovdEfu\_yx$, $CovdEfu\_yy$, $CovdEfu\_yz$, $CovdEfu\_zx$, $CovdEfu\_zy$, $CovdEfu\_zz$, $CovdEfd\_xx$, $CovdEfd\_xy$, $CovdEfd\_xz$, $CovdEfd\_yx$, $CovdEfd\_yy$, $CovdEfd\_yz$, $CovdEfd\_zx$, $CovdEfd\_zy$, $CovdEfd\_zz$, $CovuEfu\_xx$, $CovuEfu\_xy$, $CovuEfu\_xz$, $CovuEfu\_yx$, $CovuEfu\_yy$, $CovuEfu\_yz$, $CovuEfu\_zx$, $CovuEfu\_zy$, $CovuEfu\_zz$, $Phivh\_x$, $Phivh\_y$, $Phivh\_z$, $PhiMh\_xx$, $PhiMh\_xy$, $PhiMh\_xz$, $PhiMh\_xy$, $PhiMh\_yy$, $PhiMh\_yz$, $PhiMh\_xz$, $PhiMh\_yz$, $PhiMh\_zz$, $PhiThetah$, $PhiAh$, $HPres$, $HTheta$, Hv_x, Hv_y, Hv_z, HM_xy, HM_xz, HM_yz, HE_x, HE_y, HE_z, HN_x, HN_y, HN_z, HNe_x, HNe_y, HNe_z, HT_xx, HT_xy, HT_xz, HT_yy, HT_yz, HT_zz, $Sfud\_xx$, $Sfud\_xy$, $Sfud\_xz$, $Sfud\_yx$, $Sfud\_yy$, $Sfud\_yz$, $Sfud\_zx$, $Sfud\_zy$, $Sfud\_zz$, $trSf$

Parameters

Parameter	Type	Default value
$do\_leakage$	INT	Not set
$threshold\_leakage\_vacuum$	REAL	Not set
eos_type	INT	Not set
$Betau\_x\_0$	REAL	Not set
$Betau\_y\_0$	REAL	Not set
$Betau\_z\_0$	REAL	Not set
$minTableTemperature$	REAL	Not set
$minTableEnergy$	REAL	Not set
$energyShift$	REAL	Not set
$vacuum\_ye\_beta$	REAL	Not set
$vacuum\_rho$	REAL	Not set
$vacuum\_ye$	REAL	Not set
$vacuum\_rho\_reset$	REAL	Not set
$vacuum\_P\_reset$	REAL	Not set
$vacuum\_ye\_reset$	REAL	Not set
$vacuum\_temp\_reset$	REAL	Not set
$vacuum\_tau\_reset$	REAL	Not set
$ye\_maximum$	REAL	Not set
$threshold\_Dles\_min$	REAL	Not set
$threshold\_Dles\_max$	REAL	Not set
$calculate\_les\_terms$	INT	Not set
$externalCon2Prim$	INT	Not set
ρ_0	REAL	Not set
ρ_1	REAL	Not set
ρ_2	REAL	Not set
$gamma\_0$	REAL	Not set
$gamma\_1$	REAL	Not set
$gamma\_2$	REAL	Not set
$gamma\_3$	REAL	Not set
a_0	REAL	3 Not set
a_1	REAL	Not set
a_2	REAL	Not set
a_3	REAL	Not set
K_0	REAL	Not set
K_1	REAL	Not set

Evolution Equations

Evolution equation

$$\partial_t Sfd_x + \frac{\partial F_x^{(Sfd\_x)}}{\partial x} + \frac{\partial F_y^{(Sfd\_x)}}{\partial y} + \frac{\partial F_z^{(Sfd\_x)}}{\partial z} = S^{(Sfd\_x)} + Op(x,y,z,t) \tag{1}$$

where the fluxes and sources are:

$$\begin{aligned} F_x^{(Sfd\_x)} = &(-Betau\_x\ Sfd\_x) + Alpha\ Sfud\_xx \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xx\ gtd\_xx + TauT\_xy\ gtd\_xy \\ &\qquad + TauT\_xz\ gtd\_xz)) \end{aligned} \tag{2}$$

$$\begin{aligned} F_y^{(Sfd\_x)} = &(-Betau\_y\ Sfd\_x) + Alpha\ Sfud\_yx \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xy\ gtd\_xx + TauT\_yy\ gtd\_xy \\ &\qquad + TauT\_yz\ gtd\_xz)) \end{aligned} \tag{3}$$

$$\begin{aligned} F_z^{(Sfd\_x)} = &(-Betau\_z\ Sfd\_x) + Alpha\ Sfud\_zx \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xz\ gtd\_xx + TauT\_yz\ gtd\_xy \\ &\qquad + TauT\_zz\ gtd\_xz)) \end{aligned} \tag{4}$$

$$S^{(Sfd\_x)} = 0 \tag{5}$$

$$\begin{aligned} Op(x,y,z,t) = &+0.5\ \frac{Alpha\ Sfuu\_xx}{chi\_max}\ \partial_x gtd\_xx + \frac{Alpha\ Sfuu\_xy}{chi\_max}\ \partial_x gtd\_xy \\ &+ \frac{Alpha\ Sfuu\_xz}{chi\_max}\ \partial_x gtd\_xz + 0.5\ \frac{Alpha\ Sfuu\_yy}{chi\_max}\ \partial_x gtd\_yy \\ &+ \frac{Alpha\ Sfuu\_yz}{chi\_max}\ \partial_x gtd\_yz + 0.5\ \frac{Alpha\ Sfuu\_zz}{chi\_max}\ \partial_x gtd\_zz \\ &- 0.5\ \frac{Alpha\ trSf}{chi\_max}\ \partial_x chi + Sfd\_x\ \partial_x Betau\_x \\ &+ Sfd\_y\ \partial_x Betau\_y + Sfd\_z\ \partial_x Betau\_z \\ &- (Df + tauf)\ \partial_x Alpha + Alpha\ sdetg\ sqW\ qnu\ vfu\_x \end{aligned} \tag{6}$$

Evolution equation

$$\partial_t Sfd_y + \frac{\partial F_x^{(Sfd\_y)}}{\partial x} + \frac{\partial F_y^{(Sfd\_y)}}{\partial y} + \frac{\partial F_z^{(Sfd\_y)}}{\partial z} = S^{(Sfd\_y)} + Op(x,y,z,t) \tag{7}$$

where the fluxes and sources are:

$$\begin{aligned} F_x^{(Sfd\_y)} = &(-Betau\_x\ Sfd\_y) + Alpha\ Sfud\_xy \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xx\ gtd\_xy + TauT\_xy\ gtd\_yy \\ &\qquad + TauT\_xz\ gtd\_yz)) \end{aligned} \tag{8}$$

$$\begin{aligned} F_y^{(Sfd\_y)} = &(-Betau\_y\ Sfd\_y) + Alpha\ Sfud\_yy \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xy\ gtd\_xy + TauT\_yy\ gtd\_yy \\ &\qquad + TauT\_yz\ gtd\_yz)) \end{aligned} \tag{9}$$

$$\begin{aligned} F_z^{(Sfd\_y)} = &(-Betau\_z\ Sfd\_y) + Alpha\ Sfud\_zy \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xz\ gtd\_xy + TauT\_yz\ gtd\_yy \\ &\qquad + TauT\_zz\ gtd\_yz)) \end{aligned} \tag{10}$$

$$S^{(Sfd\_y)} = 0 \tag{11}$$

$$\begin{aligned} Op(x,y,z,t) = &+0.5\ \frac{Alpha\ Sfuu\_xx}{chi\_max}\ \partial_y gtd\_xx + \frac{Alpha\ Sfuu\_xy}{chi\_max}\ \partial_y gtd\_xy \\ &+ \frac{Alpha\ Sfuu\_xz}{chi\_max}\ \partial_y gtd\_xz + 0.5\ \frac{Alpha\ Sfuu\_yy}{chi\_max}\ \partial_y gtd\_yy \\ &+ \frac{Alpha\ Sfuu\_yz}{chi\_max}\ \partial_y gtd\_yz + 0.5\ \frac{Alpha\ Sfuu\_zz}{chi\_max}\ \partial_y gtd\_zz \\ &- 0.5\ \frac{Alpha\ trSf}{chi\_max}\ \partial_y chi + Sfd\_x\ \partial_y Betau\_x \\ &+ Sfd\_y\ \partial_y Betau\_y + Sfd\_z\ \partial_y Betau\_z \\ &- (Df + tauf)\ \partial_y Alpha + Alpha\ sdetg\ sqW\ qnu\ vfu\_y \end{aligned} \tag{12}$$

Evolution equation

$$\partial_t Sfd_z + \frac{\partial F_x^{(Sfd\_z)}}{\partial x} + \frac{\partial F_y^{(Sfd\_z)}}{\partial y} + \frac{\partial F_z^{(Sfd\_z)}}{\partial z} = S^{(Sfd\_z)} + Op(x,y,z,t) \tag{13}$$

where the fluxes and sources are:

$$\begin{aligned} F_x^{(Sfd\_z)} = &(-Betau\_x\ Sfd\_z) + Alpha\ Sfud\_xz \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xx\ gtd\_xz + TauT\_xy\ gtd\_yz \\ &\qquad + TauT\_xz\ gtd\_zz)) \end{aligned} \tag{14}$$

$$\begin{aligned} F_y^{(Sfd\_z)} = &(-Betau\_y\ Sfd\_z) + Alpha\ Sfud\_yz \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xy\ gtd\_xz + TauT\_yy\ gtd\_yz \\ &\qquad + TauT\_yz\ gtd\_zz)) \end{aligned} \tag{15}$$

$$\begin{aligned} F_z^{(Sfd\_z)} = &(-Betau\_z\ Sfd\_z) + Alpha\ Sfud\_zz \\ &+ (-Alpha\ sdetg\ inv\_chi\ \ (TauT\_xz\ gtd\_xz + TauT\_yz\ gtd\_yz \\ &\qquad + TauT\_zz\ gtd\_zz)) \end{aligned} \tag{16}$$

$$S^{(Sfd\_z)} = 0 \tag{17}$$

$$\begin{aligned} Op(x,y,z,t) = &+0.5\ \frac{Alpha\ Sfuu\_xx}{chi\_max}\ \partial_z gtd\_xx + \frac{Alpha\ Sfuu\_xy}{chi\_max}\ \partial_z gtd\_xy \\ &+ \frac{Alpha\ Sfuu\_xz}{chi\_max}\ \partial_z gtd\_xz + 0.5\ \frac{Alpha\ Sfuu\_yy}{chi\_max}\ \partial_z gtd\_yy \\ &+ \frac{Alpha\ Sfuu\_yz}{chi\_max}\ \partial_z gtd\_yz + 0.5\ \frac{Alpha\ Sfuu\_zz}{chi\_max}\ \partial_z gtd\_zz \\ &- 0.5\ \frac{Alpha\ trSf}{chi\_max}\ \partial_z chi + Sfd\_x\ \partial_z Betau\_x \\ &+ Sfd\_y\ \partial_z Betau\_y + Sfd\_z\ \partial_z Betau\_z \\ &- (Df + tauf)\ \partial_z Alpha + Alpha\ sdetg\ sqW\ qnu\ vfu\_z \end{aligned} \tag{18}$$

Evolution equation

$$\partial_t Bfu_x + \frac{\partial F_x^{(Bfu\_x)}}{\partial x} + \frac{\partial F_y^{(Bfu\_x)}}{\partial y} + \frac{\partial F_z^{(Bfu\_x)}}{\partial z} = S^{(Bfu\_x)} + Op(x,y,z,t) \quad (19)$$

where the fluxes and sources are:

$$F_x^{(Bfu\_x)} = Alpha\ chi\_max\ gtu\_xx\ phif \quad (20)$$

$$\begin{aligned} F_y^{(Bfu\_x)} = {} & Bfu\_x\ \ (Alpha\ vfu\_y - Betau\_y) + (-Bfu\_y\ \ (Alpha\ vfu\_x - Betau\_x)) \\ & + Alpha\ chi\_max\ gtu\_xy\ phif + Alpha\ sdetg\ TauM\_xy \end{aligned} \quad (21)$$

$$\begin{aligned} F_z^{(Bfu\_x)} = {} & Bfu\_x\ \ (Alpha\ vfu\_z - Betau\_z) + (-Bfu\_z\ \ (Alpha\ vfu\_x - Betau\_x)) \\ & + Alpha\ chi\_max\ gtu\_xz\ phif + Alpha\ sdetg\ TauM\_xz \end{aligned} \quad (22)$$

$$S^{(Bfu\_x)} = -phif\ \ Alpha\ chi\_max\ Gamt\_x \quad (23)$$

$$\begin{aligned} Op(x,y,z,t) = {} & -0.5\ phif\ gtu\_xx\ Alpha\ \partial_x chi - 0.5\ phif\ gtu\_xy\ Alpha\ \partial_y chi \\ & - 0.5\ phif\ gtu\_xz\ Alpha\ \partial_z chi + phif\ gtu\_xx\ chi\_max\ \partial_x Alpha \\ & + phif\ gtu\_xy\ chi\_max\ \partial_y Alpha + phif\ gtu\_xz\ chi\_max\ \partial_z Alpha \end{aligned} \quad (24)$$

Evolution equation

$$\partial_t Bfu_y + \frac{\partial F_x^{(Bfu\_y)}}{\partial x} + \frac{\partial F_y^{(Bfu\_y)}}{\partial y} + \frac{\partial F_z^{(Bfu\_y)}}{\partial z} = S^{(Bfu\_y)} + Op(x,y,z,t) \quad (25)$$

where the fluxes and sources are:

$$\begin{aligned} F_x^{(Bfu\_y)} = {} & Bfu\_y\ \ (Alpha\ vfu\_x - Betau\_x) + (-Bfu\_x\ \ (Alpha\ vfu\_y - Betau\_y)) \\ & + Alpha\ chi\_max\ gtu\_xy\ phif + (-Alpha\ sdetg\ TauM\_xy) \end{aligned} \quad (26)$$

$$F_y^{(Bfu\_y)} = Alpha\ chi\_max\ gtu\_yy\ phif \quad (27)$$

$$F_z^{(Bfu\_y)} = Bfu\_y\ \ (Alpha\ vfu\_z - Betau\_z) + (-Bfu\_z\ \ (Alpha\ vfu\_y - Betau\_y)) + Alpha\ chi\_max\ gtu\_yz\ phif + Alpha\ sdetg\ TauM\_yz \tag{28}$$

$$S^{(Bfu\_y)} = -phif\ Alpha\ chi\_max\ Gamt\_y \tag{29}$$

$$Op(x,y,z,t) = -0.5\ phif\ gtu\_xy\ Alpha\ \partial_x chi - 0.5\ phif\ gtu\_yy\ Alpha\ \partial_y chi - 0.5\ phif\ gtu\_yz\ Alpha\ \partial_z chi + phif\ gtu\_xy\ chi\_max\ \partial_x Alpha + phif\ gtu\_yy\ chi\_max\ \partial_y Alpha + phif\ gtu\_yz\ chi\_max\ \partial_z Alpha \tag{30}$$

Evolution equation

$$\partial_t Bfu_z + \frac{\partial F_x^{(Bfu\_z)}}{\partial x} + \frac{\partial F_y^{(Bfu\_z)}}{\partial y} + \frac{\partial F_z^{(Bfu\_z)}}{\partial z} = S^{(Bfu\_z)} + Op(x,y,z,t) \tag{31}$$

where the fluxes and sources are:

$$F_x^{(Bfu\_z)} = Bfu\_z\ \ (Alpha\ vfu\_x - Betau\_x) + (-Bfu\_x\ \ (Alpha\ vfu\_z - Betau\_z)) + Alpha\ chi\_max\ gtu\_xz\ phif + (-Alpha\ sdetg\ TauM\_xz) \tag{32}$$

$$F_y^{(Bfu\_z)} = Bfu\_z\ \ (Alpha\ vfu\_y - Betau\_y) + (-Bfu\_y\ \ (Alpha\ vfu\_z - Betau\_z)) + Alpha\ chi\_max\ gtu\_yz\ phif + (-Alpha\ sdetg\ TauM\_yz) \tag{33}$$

$$F_z^{(Bfu\_z)} = Alpha\ chi\_max\ gtu\_zz\ phif \tag{34}$$

$$S^{(Bfu\_z)} = -phif\ Alpha\ chi\_max\ Gamt\_z \tag{35}$$

$$Op(x,y,z,t) = -0.5\ phif\ gtu\_xz\ Alpha\ \partial_x chi - 0.5\ phif\ gtu\_yz\ Alpha\ \partial_y chi - 0.5\ phif\ gtu\_zz\ Alpha\ \partial_z chi + phif\ gtu\_xz\ chi\_max\ \partial_x Alpha + phif\ gtu\_yz\ chi\_max\ \partial_y Alpha + phif\ gtu\_zz\ chi\_max\ \partial_z Alpha \tag{36}$$

Evolution equation

$$\partial_t Df + \frac{\partial F_x^{(Df)}}{\partial x} + \frac{\partial F_y^{(Df)}}{\partial y} + \frac{\partial F_z^{(Df)}}{\partial z} = S^{(Df)} \tag{37}$$

where the fluxes and sources are:

$$F_x^{(Df)} = (Alpha\ vfu\_x - Betau\_x)\ \ Df + (-Alpha\ sdetg\ TauN\_x) \tag{38}$$

$$F_y^{(Df)} = (Alpha\ vfu\_y - Betau\_y)\ \ Df + (-Alpha\ sdetg\ TauN\_y) \tag{39}$$

$$F_z^{(Df)} = (Alpha\ vfu\_z - Betau\_z)\ \ Df + (-Alpha\ sdetg\ TauN\_z) \tag{40}$$

$$S^{(Df)} = \mathrm{o} \tag{41}$$

Evolution equation

$$\partial_t DYf + \frac{\partial F_x^{(DYf)}}{\partial x} + \frac{\partial F_y^{(DYf)}}{\partial y} + \frac{\partial F_z^{(DYf)}}{\partial z} = S^{(DYf)} + Op(x,y,z,t) \tag{42}$$

where the fluxes and sources are:

$$F_x^{(DYf)} = (Alpha\ vfu\_x - Betau\_x)\ \ DYf + (-Alpha\ sdetg\ TauNe\_x) \tag{43}$$

$$F_y^{(DYf)} = (Alpha\ vfu\_y - Betau\_y)\ \ DYf + (-Alpha\ sdetg\ TauNe\_y) \tag{44}$$

$$F_z^{(DYf)} = (Alpha\ vfu\_z - Betau\_z)\ \ DYf + (-Alpha\ sdetg\ TauNe\_z) \tag{45}$$

$$S^{(DYf)} = \mathrm{o} \tag{46}$$

$$Op(x,y,z,t) = +Alpha\ sdetg\ rnu \tag{47}$$

Evolution equation

$$\partial_t tauf + \frac{\partial F_x^{(tauf)}}{\partial x} + \frac{\partial F_y^{(tauf)}}{\partial y} + \frac{\partial F_z^{(tauf)}}{\partial z} = S^{(tauf)} + Op(x,y,z,t) \tag{48}$$

where the fluxes and sources are:

$$F_x^{(tauf)} = (-Betau\_x\ tauf) + Alpha\ \ ((-Df\ vfu\_x) + Sfu\_x) + Alpha\ sdetg\ TauN\_x \tag{49}$$

$$F_y^{(tauf)} = (-Betau\_y\ tauf) + Alpha\ \ ((-Df\ vfu\_y) + Sfu\_y) + Alpha\ sdetg\ TauN\_y \tag{50}$$

$$F_z^{(tauf)} = (-Betau\_z\ tauf) + Alpha\ \ ((-Df\ vfu\_z) + Sfu\_z) + Alpha\ sdetg\ TauN\_z \tag{51}$$

$$S^{(tauf)} \tag{52}$$
$$= \frac{Alpha\ \ (Sfuu\_xx\ Atd\_xx + 2\ Sfuu\_xy\ Atd\_xy + 2\ Sfuu\_xz\ Atd\_xz + Sfuu\_yy\ Atd\_yy + 2\ Sfuu\_yz\ Atd\_}{chi\_max}$$
$$+ 0.3333333333333333\ Alpha\ trSf\ trK$$

$$Op(x,y,z,t) = -Sfu\_x\ \partial_x Alpha - Sfu\_y\ \partial_y Alpha - Sfu\_z\ \partial_z Alpha + Alpha\ sdetg\ sqW\ qnu \tag{53}$$

Evolution equation

$$\partial_t phif + \frac{\partial F_x^{(phif)}}{\partial x} + \frac{\partial F_y^{(phif)}}{\partial y} + \frac{\partial F_z^{(phif)}}{\partial z} = S^{(phif)} + Op(x,y,z,t) \tag{54}$$

where the fluxes and sources are:

$$F_x^{(phif)} = Alpha\ ch^2\ Bfu\_x + (-phif\ Betau\_x) \tag{55}$$

$$F_y^{(phif)} = Alpha\ ch^2\ Bfu\_y + (-phif\ Betau\_y) \tag{56}$$

$$F_z^{(phif)} = Alpha\ ch^2\ Bfu\_z + (-phif\ Betau\_z) \tag{57}$$

$$S^{(phif)} = (-Alpha\ kappa\_f\ phif) + (-Alpha\ phif\ trK) \tag{58}$$

$$Op(x,y,z,t) = +ch^2\ Bfu\_x\ \partial_x Alpha + ch^2\ Bfu\_y\ \partial_y Alpha + ch^2\ Bfu\_z\ \partial_z Alpha \tag{59}$$

Evolution equation

$$\partial_t gtd_x x = S^{(gtd\_xx)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{60}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +2\ gtd\_xx\ \partial_x Betau\_x + 2\ gtd\_xy\ \partial_x Betau\_y \\ & + 2\ gtd\_xz\ \partial_x Betau\_z + (-0.6666666666666667)\ gtd\_xx\ div\_Beta \\ & + (-2.0)\ Alpha\ ((-0.3333333333333333\ trAt\ lambda\_0\ gtd\_xx) \\ & \qquad + Atd\_xx) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_xx\ \ln detgtd \end{aligned} \tag{61}$$

$$\begin{aligned} forward(x,y,z,t) = {} & + \max\{0, Betau\_x\}\ \partial_x gtd\_xx \\ & + \max\{0, Betau\_y\}\ \partial_y gtd\_xx + \max\{0, Betau\_z\}\ \partial_z gtd\_xx \end{aligned} \tag{62}$$

$$\begin{aligned} backward(x,y,z,t) = {} & + \min\{0, Betau\_x\}\ \partial_x gtd\_xx \\ & + \min\{0, Betau\_y\}\ \partial_y gtd\_xx + \min\{0, Betau\_z\}\ \partial_z gtd\_xx \end{aligned} \tag{63}$$

Evolution equation

$$\partial_t gtd_x y = S^{(gtd\_xy)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{64}$$

$$
\begin{aligned}
Op(x,y,z,t) = &+gtd\_xx\ \partial_y Betau\_x + gtd\_xy\ \partial_y Betau\_y + gtd\_xz\ \partial_y Betau\_z \\
&+ gtd\_xy\ \partial_x Betau\_x + gtd\_yy\ \partial_x Betau\_y + gtd\_yz\ \partial_x Betau\_z \\
&+ (-0.6666666666666667)\ gtd\_xy\ div\_Beta \\
&+ (-2.0)\ Alpha\ ((-0.3333333333333333\ trAt\ lambda\_0\ gtd\_xy) \\
&\qquad + Atd\_xy) \\
&+ (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_xy\ \ln detgtd
\end{aligned}
\tag{65}
$$

$$
\begin{aligned}
forward(x,y,z,t) = &+ \max\{0, Betau\_x\}\ \partial_x gtd\_xy \\
&+ \max\{0, Betau\_y\}\ \partial_y gtd\_xy + \max\{0, Betau\_z\}\ \partial_z gtd\_xy
\end{aligned}
\tag{66}
$$

$$
\begin{aligned}
backward(x,y,z,t) = &+ \min\{0, Betau\_x\}\ \partial_x gtd\_xy \\
&+ \min\{0, Betau\_y\}\ \partial_y gtd\_xy + \min\{0, Betau\_z\}\ \partial_z gtd\_xy
\end{aligned}
\tag{67}
$$

Evolution equation

$$
\partial_t gtd_x z = S^{(gtd\_xz)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t)
\tag{68}
$$

$$
\begin{aligned}
Op(x,y,z,t) = &+gtd\_xx\ \partial_z Betau\_x + gtd\_xy\ \partial_z Betau\_y + gtd\_xz\ \partial_z Betau\_z \\
&+ gtd\_xz\ \partial_x Betau\_x + gtd\_yz\ \partial_x Betau\_y + gtd\_zz\ \partial_x Betau\_z \\
&+ (-0.6666666666666667)\ gtd\_xz\ div\_Beta \\
&+ (-2.0)\ Alpha\ ((-0.3333333333333333\ trAt\ lambda\_0\ gtd\_xz) \\
&\qquad + Atd\_xz) \\
&+ (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_xz\ \ln detgtd
\end{aligned}
\tag{69}
$$

$$
\begin{aligned}
forward(x,y,z,t) = &+ \max\{0, Betau\_x\}\ \partial_x gtd\_xz + \max\{0, Betau\_y\}\ \partial_y gtd\_xz \\
&+ \max\{0, Betau\_z\}\ \partial_z gtd\_xz
\end{aligned}
\tag{70}
$$

$$
\begin{aligned}
backward(x,y,z,t) = &+ \min\{0, Betau\_x\}\ \partial_x gtd\_xz \\
&+ \min\{0, Betau\_y\}\ \partial_y gtd\_xz + \min\{0, Betau\_z\}\ \partial_z gtd\_xz
\end{aligned}
\tag{71}
$$

Evolution equation

$$\partial_t gtd_y y = S^{(gtd\_yy)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{72}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +2\ gtd\_xy\ \partial_y Betau\_x + 2\ gtd\_yy\ \partial_y Betau\_y \\ & + 2\ gtd\_yz\ \partial_y Betau\_z + (-0.6666666666666667)\ gtd\_yy\ div\_Beta \\ & + (-2.0)\ Alpha\ ((-0.3333333333333333\ trAt\ lambda\_0\ gtd\_yy) \\ & \qquad + Atd\_yy) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_yy\ \ln detgtd \end{aligned} \tag{73}$$

$$\begin{aligned} forward(x,y,z,t) = {} & + \max\{0, Betau\_x\}\ \partial_x gtd\_yy + \max\{0, Betau\_y\}\ \partial_y gtd\_yy \\ & + \max\{0, Betau\_z\}\ \partial_z gtd\_yy \end{aligned} \tag{74}$$

$$\begin{aligned} backward(x,y,z,t) = {} & + \min\{0, Betau\_x\}\ \partial_x gtd\_yy \\ & + \min\{0, Betau\_y\}\ \partial_y gtd\_yy + \min\{0, Betau\_z\}\ \partial_z gtd\_yy \end{aligned} \tag{75}$$

Evolution equation

$$\partial_t gtd_y z = S^{(gtd\_yz)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{76}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +gtd\_xy\ \partial_z Betau\_x + gtd\_yy\ \partial_z Betau\_y + gtd\_yz\ \partial_z Betau\_z \\ & + gtd\_xz\ \partial_y Betau\_x + gtd\_yz\ \partial_y Betau\_y + gtd\_zz\ \partial_y Betau\_z \\ & + (-0.6666666666666667)\ gtd\_yz\ div\_Beta \\ & + (-2.0)\ Alpha\ ((-0.3333333333333333\ trAt\ lambda\_0\ gtd\_yz) \\ & \qquad + Atd\_yz) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_yz\ \ln detgtd \end{aligned} \tag{77}$$

$$\begin{aligned} forward(x,y,z,t) = {} & + \max\{0, Betau\_x\}\ \partial_x gtd\_yz \\ & + \max\{0, Betau\_y\}\ \partial_y gtd\_yz + \max\{0, Betau\_z\}\ \partial_z gtd\_yz \end{aligned} \tag{78}$$

$$backward(x,y,z,t) = +\min\{0, Betau\_x\}\ \partial_x gtd\_yz + \min\{0, Betau\_y\}\ \partial_y gtd\_yz + \min\{0, Betau\_z\}\ \partial_z gtd\_yz \tag{79}$$

Evolution equation

$$\partial_t gtd_z z = S^{(gtd\_zz)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{80}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +2\ gtd\_xz\ \partial_z Betau\_x + 2\ gtd\_yz\ \partial_z Betau\_y \\ & + 2\ gtd\_zz\ \partial_z Betau\_z + (-0.6666666666666667)\ gtd\_zz\ div\_Beta \\ & + (-2.0)\ Alpha\ ((-0.3333333333333333\ trAt\ lambda\_0\ gtd\_zz) \\ & \qquad + Atd\_zz) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_zz\ \ln detgtd \end{aligned} \tag{81}$$

$$forward(x,y,z,t) = +\max\{0, Betau\_x\}\ \partial_x gtd\_zz + \max\{0, Betau\_y\}\ \partial_y gtd\_zz + \max\{0, Betau\_z\}\ \partial_z gtd\_zz \tag{82}$$

$$backward(x,y,z,t) = +\min\{0, Betau\_x\}\ \partial_x gtd\_zz + \min\{0, Betau\_y\}\ \partial_y gtd\_zz + \min\{0, Betau\_z\}\ \partial_z gtd\_zz \tag{83}$$

Evolution equation

$$\partial_t Atd_x x = S^{(Atd\_xx)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{84}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +2\ Atd\_xx\ \partial_x Betau\_x + 2\ Atd\_xy\ \partial_x Betau\_y + 2\ Atd\_xz\ \partial_x Betau\_z \\ & + (-0.6666666666666667)\ Atd\_xx\ div\_Beta + Psi1TF\_xx \\ & + Alpha\ (trK\ Atd\_xx + (-2.0\ Atd\_xx\ Atud\_xx) \\ & \qquad + (-2.0\ Atd\_xy\ Atud\_yx) + (-2.0\ Atd\_xz\ Atud\_zx)) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_xx\ trAt \end{aligned} \tag{85}$$

$$forward(x,y,z,t) = +\max\{0, Betau\_x\}\ \partial_x Atd\_xx + \max\{0, Betau\_y\}\ \partial_y Atd\_xx + \max\{0, Betau\_z\}\ \partial_z Atd\_xx \tag{86}$$

$$backward(x,y,z,t) = +\min\{0, Betau\_x\}\ \partial_x Atd\_xx + \min\{0, Betau\_y\}\ \partial_y Atd\_xx + \min\{0, Betau\_z\}\ \partial_z Atd\_xx \tag{87}$$

Evolution equation

$$\partial_t Atd_x y = S^{(Atd\_xy)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{88}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +Atd\_xx\ \partial_y Betau\_x + Atd\_xy\ \partial_y Betau\_y + Atd\_xz\ \partial_y Betau\_z \\ & + Atd\_xy\ \partial_x Betau\_x + Atd\_yy\ \partial_x Betau\_y + Atd\_yz\ \partial_x Betau\_z \\ & + (-0.6666666666666667)\ Atd\_xy\ div\_Beta + Psi1TF\_xy \\ & + Alpha\ \ (trK\ Atd\_xy + (-2.0\ Atd\_xx\ Atud\_xy) \\ & \qquad + (-2.0\ Atd\_xy\ Atud\_yy) + (-2.0\ Atd\_xz\ Atud\_zy)) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_xy\ trAt \end{aligned} \tag{89}$$

$$forward(x,y,z,t) = +\max\{0, Betau\_x\}\ \partial_x Atd\_xy + \max\{0, Betau\_y\}\ \partial_y Atd\_xy + \max\{0, Betau\_z\}\ \partial_z Atd\_xy \tag{90}$$

$$backward(x,y,z,t) = +\min\{0, Betau\_x\}\ \partial_x Atd\_xy + \min\{0, Betau\_y\}\ \partial_y Atd\_xy + \min\{0, Betau\_z\}\ \partial_z Atd\_xy \tag{91}$$

Evolution equation

$$\partial_t Atd_x z = S^{(Atd\_xz)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{92}$$

$$
\begin{aligned}
Op(x,y,z,t) = &+ Atd\_xx\ \partial_z Betau\_x + Atd\_xy\ \partial_z Betau\_y + Atd\_xz\ \partial_z Betau\_z \\
&+ Atd\_xz\ \partial_x Betau\_x + Atd\_yz\ \partial_x Betau\_y + Atd\_zz\ \partial_x Betau\_z \\
&+ (-0.6666666666666667)\ Atd\_xz\ div\_Beta + Psi1TF\_xz \\
&+ Alpha\ (trK\ Atd\_xz + (-2.0\ Atd\_xx\ Atud\_xz) \\
&\qquad + (-2.0\ Atd\_xy\ Atud\_yz) + (-2.0\ Atd\_xz\ Atud\_zz)) \\
&+ (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_xz\ trAt
\end{aligned}
\tag{93}
$$

$$
\begin{aligned}
forward(x,y,z,t) = &+ \max\{0, Betau\_x\}\ \partial_x Atd\_xz \\
&+ \max\{0, Betau\_y\}\ \partial_y Atd\_xz + \max\{0, Betau\_z\}\ \partial_z Atd\_xz
\end{aligned}
\tag{94}
$$

$$
\begin{aligned}
backward(x,y,z,t) = &+ \min\{0, Betau\_x\}\ \partial_x Atd\_xz \\
&+ \min\{0, Betau\_y\}\ \partial_y Atd\_xz + \min\{0, Betau\_z\}\ \partial_z Atd\_xz
\end{aligned}
\tag{95}
$$

Evolution equation

$$
\partial_t Atd_y y = S^{(Atd\_yy)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t)
\tag{96}
$$

$$
\begin{aligned}
Op(x,y,z,t) = &+ 2\ Atd\_xy\ \partial_y Betau\_x + 2\ Atd\_yy\ \partial_y Betau\_y + 2\ Atd\_yz\ \partial_y Betau\_z \\
&+ (-0.6666666666666667)\ Atd\_yy\ div\_Beta + Psi1TF\_yy \\
&+ Alpha\ (trK\ Atd\_yy + (-2.0\ Atd\_xy\ Atud\_xy) \\
&\qquad + (-2.0\ Atd\_yy\ Atud\_yy) + (-2.0\ Atd\_yz\ Atud\_zy)) \\
&+ (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_yy\ trAt
\end{aligned}
\tag{97}
$$

$$
\begin{aligned}
forward(x,y,z,t) = &+ \max\{0, Betau\_x\}\ \partial_x Atd\_yy \\
&+ \max\{0, Betau\_y\}\ \partial_y Atd\_yy + \max\{0, Betau\_z\}\ \partial_z Atd\_yy
\end{aligned}
\tag{98}
$$

$$
\begin{aligned}
backward(x,y,z,t) = &+ \min\{0, Betau\_x\}\ \partial_x Atd\_yy \\
&+ \min\{0, Betau\_y\}\ \partial_y Atd\_yy + \min\{0, Betau\_z\}\ \partial_z Atd\_yy
\end{aligned}
\tag{99}
$$

Evolution equation

$$\partial_t Atd_y z = S^{(Atd\_yz)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{100}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +Atd\_xy\ \partial_z Betau\_x + Atd\_yy\ \partial_z Betau\_y + Atd\_yz\ \partial_z Betau\_z \\ & + Atd\_xz\ \partial_y Betau\_x + Atd\_yz\ \partial_y Betau\_y + Atd\_zz\ \partial_y Betau\_z \\ & + (-0.6666666666666667)\ Atd\_yz\ div\_Beta + Psi1TF\_yz \\ & + Alpha\ (trK\ Atd\_yz + (-2.0\ Atd\_xy\ Atud\_xz) \\ & \qquad + (-2.0\ Atd\_yy\ Atud\_yz) + (-2.0\ Atd\_yz\ Atud\_zz)) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_yz\ trAt \end{aligned} \tag{101}$$

$$\begin{aligned} forward(x,y,z,t) = {} & +\max\{0, Betau\_x\}\ \partial_x Atd\_yz \\ & + \max\{0, Betau\_y\}\ \partial_y Atd\_yz + \max\{0, Betau\_z\}\ \partial_z Atd\_yz \end{aligned} \tag{102}$$

$$\begin{aligned} backward(x,y,z,t) = {} & +\min\{0, Betau\_x\}\ \partial_x Atd\_yz \\ & + \min\{0, Betau\_y\}\ \partial_y Atd\_yz + \min\{0, Betau\_z\}\ \partial_z Atd\_yz \end{aligned} \tag{103}$$

Evolution equation

$$\partial_t Atd_z z = S^{(Atd\_zz)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{104}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +2\ Atd\_xz\ \partial_z Betau\_x + 2\ Atd\_yz\ \partial_z Betau\_y + 2\ Atd\_zz\ \partial_z Betau\_z \\ & + (-0.6666666666666667)\ Atd\_zz\ div\_Beta + Psi1TF\_zz \\ & + Alpha\ (trK\ Atd\_zz + (-2.0\ Atd\_xz\ Atud\_xz) \\ & \qquad + (-2.0\ Atd\_yz\ Atud\_yz) + (-2.0\ Atd\_zz\ Atud\_zz)) \\ & + (-0.3333333333333333)\ kappa\_cc\ Alpha\ gtd\_zz\ trAt \end{aligned} \tag{105}$$

$$\begin{aligned} forward(x,y,z,t) = {} & +\max\{0, Betau\_x\}\ \partial_x Atd\_zz \\ & + \max\{0, Betau\_y\}\ \partial_y Atd\_zz + \max\{0, Betau\_z\}\ \partial_z Atd\_zz \end{aligned} \tag{106}$$

$$backward(x,y,z,t) = +\min\{0, Betau\_x\}\ \partial_x Atd\_zz + \min\{0, Betau\_y\}\ \partial_y Atd\_zz + \min\{0, Betau\_z\}\ \partial_z Atd\_zz \tag{107}$$

Evolution equation

$$\partial_t Gamh_x = S^{(Gamh\_x)} + Op(x,y,z,t) + backward(x,y,z,t) + forward(x,y,z,t) \tag{108}$$

$$\begin{aligned} Op(x,y,z,t) = {} & +0.6666666666666667\ Gamh\_x\ div\_Beta \\ & + 2.0\ Alpha\ (Ct\_xxx\ Atu\_xx + 2\ Ct\_xxy\ Atu\_xy + 2\ Ct\_xxz\ Atu\_xz \\ & \quad + Ct\_xyy\ Atu\_yy + 2\ Ct\_xyz\ Atu\_yz + Ct\_xzz\ Atu\_zz) \\ & - 1.333333333333333\ Alpha\ gtu\_xx\ \partial_x trK \\ & - 1.333333333333333\ Alpha\ gtu\_xy\ \partial_y trK \\ & - 1.333333333333333\ Alpha\ gtu\_xz\ \partial_z trK \\ & - 0.6666666666666667\ Alpha\ gtu\_xx\ \partial_x theta \\ & - 0.6666666666666667\ Alpha\ gtu\_xy\ \partial_y theta \\ & - 0.6666666666666667\ Alpha\ gtu\_xz\ \partial_z theta \\ & - 2.0\ theta\ gtu\_xx\ \partial_x Alpha \\ & - 2.0\ theta\ gtu\_xy\ \partial_y Alpha - 2.0\ theta\ gtu\_xz\ \partial_z Alpha \\ & + (-50.26548245743669)\ Alpha\ inv\_chi\ (gtu\_xx\ Jtd\_ADM\_x \\ & + gtu\_xy\ Jtd\_ADM\_y + gtu\_xz\ Jtd\_ADM\_z) - Gamh\_x\ \partial_x Betau\_x \\ & - Gamh\_y\ \partial_y Betau\_x - Gamh\_z\ \partial_z Betau\_x + gtu\_xx\ \partial_x \partial_x Betau\_x \\ & + 2\ gtu\_xy\ \partial_y \partial_x Betau\_x + 2\ gtu\_xz\ \partial_z \partial_x Betau\_x \\ & + gtu\_yy\ \partial_y \partial_y Betau\_x + 2\ gtu\_yz\ \partial_z \partial_y Betau\_x \\ & + gtu\_zz\ \partial_z \partial_z Betau\_x + 0.3333333333333333\ gtu\_xx\ d\_div\_Beta\_x \\ & + 0.3333333333333333\ gtu\_xy\ d\_div\_Beta\_y \\ & + 0.3333333333333333\ gtu\_xz\ d\_div\_Beta\_z \\ & - 2.0\ Atu\_xx\ \partial_x Alpha - 2.0\ Atu\_xy\ \partial_y Alpha \\ & - 2.0\ Atu\_xz\ \partial_z Alpha - 3.0\ Alpha\ inv\_chi\ Atu\_xx\ \partial_x chi \\ & - 3.0\ Alpha\ inv\_chi\ Atu\_xy\ \partial_y chi - 3.0\ Alpha\ inv\_chi\ Atu\_xz\ \partial_z chi \\ & + (-2.0)\ Alpha\ inv\_chi\ Zu\_x\ (kappa\_z1 + 1.333333333333333\ theta \\ & \quad + 0.6666666666666667\ trK) \end{aligned} \tag{109}$$

$$backward(x,y,z,t) = +\min\{0, Betau\_x\}\ \partial_x Gamh\_x + \min\{0, Betau\_y\}\ \partial_y Gamh\_x + \min\{0, Betau\_z\}\ \partial_z Gamh\_x \tag{110}$$

$$forward(x,y,z,t) = +\max\{0, Betau\_x\}\ \partial_x Gamh\_x + \max\{0, Betau\_y\}\ \partial_y Gamh\_x + \max\{0, Betau\_z\}\ \partial_z Gamh\_x \tag{111}$$

Evolution equation

$$\partial_t Gamh_y = S^{(Gamh\_y)} + Op(x,y,z,t) + backward(x,y,z,t) + forward(x,y,z,t) \tag{112}$$

$$
\begin{aligned}
Op(x,y,z,t) = &+0.3333333333333333\ gtu\_yz\ d\_div\_Beta\_z \\
&- 2.0\ Atu\_xy\ \partial_x Alpha - 2.0\ Atu\_yy\ \partial_y Alpha \\
&- 2.0\ Atu\_yz\ \partial_z Alpha + 0.6666666666666667\ Gamh\_y\ div\_Beta \\
&+ 2.0\ Alpha\ (Ct\_yxx\ Atu\_xx + 2\ Ct\_yxy\ Atu\_xy + 2\ Ct\_yxz\ Atu\_xz \\
&\qquad + Ct\_yyy\ Atu\_yy + 2\ Ct\_yyz\ Atu\_yz + Ct\_yzz\ Atu\_zz) \\
&- Gamh\_x\ \partial_x Betau\_y - Gamh\_y\ \partial_y Betau\_y \\
&- Gamh\_z\ \partial_z Betau\_y + gtu\_xx\ \partial_x\partial_x Betau\_y \\
&+ 2\ gtu\_xy\ \partial_y\partial_x Betau\_y + 2\ gtu\_xz\ \partial_z\partial_x Betau\_y \\
&+ gtu\_yy\ \partial_y\partial_y Betau\_y + 2\ gtu\_yz\ \partial_z\partial_y Betau\_y \\
&+ gtu\_zz\ \partial_z\partial_z Betau\_y + 0.3333333333333333\ gtu\_xy\ d\_div\_Beta\_x \\
&+ 0.3333333333333333\ gtu\_yy\ d\_div\_Beta\_y \\
&- 1.333333333333333\ Alpha\ gtu\_xy\ \partial_x trK \\
&- 1.333333333333333\ Alpha\ gtu\_yy\ \partial_y trK \\
&- 1.333333333333333\ Alpha\ gtu\_yz\ \partial_z trK \\
&- 0.6666666666666667\ Alpha\ gtu\_xy\ \partial_x theta \\
&- 0.6666666666666667\ Alpha\ gtu\_yy\ \partial_y theta \\
&- 0.6666666666666667\ Alpha\ gtu\_yz\ \partial_z theta \\
&- 2.0\ theta\ gtu\_xy\ \partial_x Alpha \\
&- 2.0\ theta\ gtu\_yy\ \partial_y Alpha - 2.0\ theta\ gtu\_yz\ \partial_z Alpha \\
&+ (-50.265482457436 69)\ Alpha\ inv\_chi\ (gtu\_xy\ Jtd\_ADM\_x \\
&\qquad + gtu\_yy\ Jtd\_ADM\_y + gtu\_yz\ Jtd\_ADM\_z) \\
&+ (-2.0)\ Alpha\ inv\_chi\ Zu\_y\ (kappa\_z1 + 1.333333333333333\ theta \\
&\qquad + 0.6666666666666667\ trK) - 3.0\ Alpha\ inv\_chi\ Atu\_xy\ \partial_x chi \\
&- 3.0\ Alpha\ inv\_chi\ Atu\_yy\ \partial_y chi - 3.0\ Alpha\ inv\_chi\ Atu\_yz\ \partial_z chi
\end{aligned}
\tag{113}
$$

$$
\begin{aligned}
backward(x,y,z,t) = &+ \min\{0, Betau\_x\}\ \partial_x Gamh\_y \\
&+ \min\{0, Betau\_y\}\ \partial_y Gamh\_y + \min\{0, Betau\_z\}\ \partial_z Gamh\_y
\end{aligned}
\tag{114}
$$

$$
\begin{aligned}
forward(x,y,z,t) = &+ \max\{0, Betau\_x\}\ \partial_x Gamh\_y + \max\{0, Betau\_y\}\ \partial_y Gamh\_y \\
&+ \max\{0, Betau\_z\}\ \partial_z Gamh\_y
\end{aligned}
\tag{115}
$$

Evolution equation

$$\partial_t Gamh_z = S^{(Gamh\_z)} + Op(x,y,z,t) + backward(x,y,z,t) + forward(x,y,z,t) \tag{116}$$

$$\begin{aligned}
Op(x,y,z,t) = &+ gtu_xx\ \partial_x\partial_x Betau_z + 2\ gtu_xy\ \partial_y\partial_x Betau_z \\
&+ 2\ gtu_xz\ \partial_z\partial_x Betau_z + gtu_yy\ \partial_y\partial_y Betau_z \\
&+ 2\ gtu_yz\ \partial_z\partial_y Betau_z + gtu_zz\ \partial_z\partial_z Betau_z \\
&+ 0.3333333333333333\ gtu_xz\ d_div_Beta_x \\
&+ 0.3333333333333333\ gtu_yz\ d_div_Beta_y \\
&+ 0.3333333333333333\ gtu_zz\ d_div_Beta_z \\
&- 2.0\ Atu_xz\ \partial_x Alpha - 2.0\ Atu_yz\ \partial_y Alpha \\
&- 2.0\ Atu_zz\ \partial_z Alpha + 0.6666666666666667\ Gamh_z\ div_Beta \\
&+ 2.0\ Alpha\ (Ct_zxx\ Atu_xx + 2\ Ct_zxy\ Atu_xy + 2\ Ct_zxz\ Atu_xz \\
&\qquad + Ct_zyy\ Atu_yy + 2\ Ct_zyz\ Atu_yz + Ct_zzz\ Atu_zz) \\
&- Gamh_x\ \partial_x Betau_z - Gamh_y\ \partial_y Betau_z - Gamh_z\ \partial_z Betau_z \\
&+ (-50.26548245743669)\ Alpha\ inv_chi\ (gtu_xz\ Jtd_ADM_x \\
&\qquad + gtu_yz\ Jtd_ADM_y + gtu_zz\ Jtd_ADM_z) \\
&- 1.333333333333333\ Alpha\ gtu_xz\ \partial_x trK \\
&- 1.333333333333333\ Alpha\ gtu_yz\ \partial_y trK \\
&- 1.333333333333333\ Alpha\ gtu_zz\ \partial_z trK \\
&- 0.6666666666666667\ Alpha\ gtu_xz\ \partial_x theta \\
&- 0.6666666666666667\ Alpha\ gtu_yz\ \partial_y theta \\
&- 0.6666666666666667\ Alpha\ gtu_zz\ \partial_z theta \\
&- 2.0\ theta\ gtu_xz\ \partial_x Alpha - 2.0\ theta\ gtu_yz\ \partial_y Alpha \\
&- 2.0\ theta\ gtu_zz\ \partial_z Alpha - 3.0\ Alpha\ inv_chi\ Atu_xz\ \partial_x chi \\
&- 3.0\ Alpha\ inv_chi\ Atu_yz\ \partial_y chi - 3.0\ Alpha\ inv_chi\ Atu_zz\ \partial_z chi \\
&+ (-2.0)\ Alpha\ inv_chi\ Zu_z\ (kappa_z1 + 1.333333333333333\ theta \\
&\qquad + 0.6666666666666667\ trK)
\end{aligned} \tag{117}$$

$$\begin{aligned}
backward(x,y,z,t) = &+ \min\{0, Betau_x\}\ \partial_x Gamh_z \\
&+ \min\{0, Betau_y\}\ \partial_y Gamh_z + \min\{0, Betau_z\}\ \partial_z Gamh_z
\end{aligned} \tag{118}$$

$$\begin{aligned}
forward(x,y,z,t) = &+ \max\{0, Betau_x\}\ \partial_x Gamh_z \\
&+ \max\{0, Betau_y\}\ \partial_y Gamh_z + \max\{0, Betau_z\}\ \partial_z Gamh_z
\end{aligned} \tag{119}$$

Evolution equation

$$\partial_t Betau_x = S^{(Betau\_x)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{120}$$

$$\begin{aligned} Op(x,y,z,t) = &+0.75\ \ (Alpha\ lambda\_f1 + lambda\_f0)\ \ Gamh\_x \\ &+ (Betau\_x\_0 - Betau\_x)\ \ feta \end{aligned} \tag{121}$$

$$\begin{aligned} forward(x,y,z,t) = &+lambda\_2\ \ \max\{0, Betau\_x\}\ \partial_x Betau\_x \\ &+ lambda\_2\ \ \max\{0, Betau\_y\}\ \partial_y Betau\_x \\ &+ lambda\_2\ \ \max\{0, Betau\_z\}\ \partial_z Betau\_x \end{aligned} \tag{122}$$

$$\begin{aligned} backward(x,y,z,t) = &+lambda\_2\ \ \min\{0, Betau\_x\}\ \partial_x Betau\_x \\ &+ lambda\_2\ \ \min\{0, Betau\_y\}\ \partial_y Betau\_x \\ &+ lambda\_2\ \ \min\{0, Betau\_z\}\ \partial_z Betau\_x \end{aligned} \tag{123}$$

Evolution equation

$$\partial_t Betau_y = S^{(Betau\_y)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{124}$$

$$\begin{aligned} Op(x,y,z,t) = &+0.75\ \ (Alpha\ lambda\_f1 + lambda\_f0)\ \ Gamh\_y \\ &+ (Betau\_y\_0 - Betau\_y)\ \ feta \end{aligned} \tag{125}$$

$$\begin{aligned} forward(x,y,z,t) = &+lambda\_2\ \ \max\{0, Betau\_x\}\ \partial_x Betau\_y \\ &+ lambda\_2\ \ \max\{0, Betau\_y\}\ \partial_y Betau\_y \\ &+ lambda\_2\ \ \max\{0, Betau\_z\}\ \partial_z Betau\_y \end{aligned} \tag{126}$$

$$\begin{aligned} backward(x,y,z,t) = &+lambda\_2\ \ \min\{0, Betau\_x\}\ \partial_x Betau\_y \\ &+ lambda\_2\ \ \min\{0, Betau\_y\}\ \partial_y Betau\_y \\ &+ lambda\_2\ \ \min\{0, Betau\_z\}\ \partial_z Betau\_y \end{aligned} \tag{127}$$

Evolution equation

$$\partial_t Betau_z = S^{(Betau\_z)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{128}$$

$$\begin{aligned} Op(x,y,z,t) = &+0.75\ \ (Alpha\ lambda\_f1 + lambda\_f0)\ \ Gamh\_z \\ &+ (Betau\_z\_0 - Betau\_z)\ \ feta \end{aligned} \tag{129}$$

$$\begin{aligned} forward(x,y,z,t) = &+lambda\_2\ \ \max\{0, Betau\_x\}\ \partial_x Betau\_z \\ &+ lambda\_2\ \ \max\{0, Betau\_y\}\ \partial_y Betau\_z \\ &+ lambda\_2\ \ \max\{0, Betau\_z\}\ \partial_z Betau\_z \end{aligned} \tag{130}$$

$$\begin{aligned} backward(x,y,z,t) = &+lambda\_2\ \ \min\{0, Betau\_x\}\ \partial_x Betau\_z \\ &+ lambda\_2\ \ \min\{0, Betau\_y\}\ \partial_y Betau\_z \\ &+ lambda\_2\ \ \min\{0, Betau\_z\}\ \partial_z Betau\_z \end{aligned} \tag{131}$$

Evolution equation

$$\partial_t Alpha = S^{(Alpha)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{132}$$

$$Op(x,y,z,t) = +(-2.0)\ \ (Alpha\ lambda\_f3 + lambda\_f2)\ \ Alpha\ \ (trK - trK0) \tag{133}$$

$$\begin{aligned} forward(x,y,z,t) = &+lambda\_1\ \ \max\{0, Betau\_x\}\ \partial_x Alpha \\ &+ lambda\_1\ \ \max\{0, Betau\_y\}\ \partial_y Alpha \\ &+ lambda\_1\ \ \max\{0, Betau\_z\}\ \partial_z Alpha \end{aligned} \tag{134}$$

$$\begin{aligned} backward(x,y,z,t) = &+lambda\_1\ \ \min\{0, Betau\_x\}\ \partial_x Alpha \\ &+ lambda\_1\ \ \min\{0, Betau\_y\}\ \partial_y Alpha \\ &+ lambda\_1\ \ \min\{0, Betau\_z\}\ \partial_z Alpha \end{aligned} \tag{135}$$

Evolution equation

$$\partial_t chi = S^{(chi)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \quad (136)$$

$$Op(x,y,z,t) = +0.6666666666666667\ chi\_max\ \ (Alpha\ \ (2.0\ theta + trK) - div\_Beta) \quad (137)$$

$$forward(x,y,z,t) = +\max\{0, Betau\_x\}\ \partial_x chi + \max\{0, Betau\_y\}\ \partial_y chi + \max\{0, Betau\_z\}\ \partial_z chi \quad (138)$$

$$backward(x,y,z,t) = +\min\{0, Betau\_x\}\ \partial_x chi + \min\{0, Betau\_y\}\ \partial_y chi + \min\{0, Betau\_z\}\ \partial_z chi \quad (139)$$

Evolution equation

$$\partial_t trK = S^{(trK)} + Op(x,y,z,t) + backward(x,y,z,t) + forward(x,y,z,t) \quad (140)$$

$$
\begin{aligned}
Op(x,y,z,t) = & -chi\_max\ gtu\_xx\ \partial_x\partial_x Alpha - chi\_max\ gtu\_yy\ \partial_y\partial_y Alpha \\
& - 2\ chi\_max\ gtu\_yz\ \partial_z\partial_y Alpha - chi\_max\ gtu\_zz\ \partial_z\partial_z Alpha \\
& + chi\_max\ Gamt\_x\ \partial_x Alpha + chi\_max\ Gamt\_y\ \partial_y Alpha \\
& + chi\_max\ Gamt\_z\ \partial_z Alpha + 0.5\ gtu\_xx\ \partial_x Alpha \partial_x chi \\
& + 0.5\ gtu\_xy\ \partial_x Alpha \partial_y chi + 0.5\ gtu\_xz\ \partial_x Alpha \partial_z chi \\
& + 0.5\ gtu\_xy\ \partial_y Alpha \partial_x chi + 0.5\ gtu\_yy\ \partial_y Alpha \partial_y chi \\
& + 0.5\ gtu\_yz\ \partial_y Alpha \partial_z chi + 0.5\ gtu\_xz\ \partial_z Alpha \partial_x chi \\
& + 0.5\ gtu\_yz\ \partial_z Alpha \partial_y chi + 0.5\ gtu\_zz\ \partial_z Alpha \partial_z chi \\
& + 4\ Alpha\ \pi\ \ (rho\_ADM + tr\_pT) \\
& - 2\ chi\_max\ gtu\_xy\ \partial_y\partial_x Alpha - 2\ chi\_max\ gtu\_xz\ \partial_z\partial_x Alpha \\
& + Alpha\ \ (Atd\_xx\ Atu\_xx + 2\ Atd\_xy\ Atu\_xy + 2\ Atd\_xz\ Atu\_xz \\
& \qquad + Atd\_yy\ Atu\_yy + 2\ Atd\_yz\ Atu\_yz + Atd\_zz\ Atu\_zz) \\
& + 2.0\ Zu\_x\ \partial_x Alpha + 2.0\ Zu\_y\ \partial_y Alpha + 2.0\ Zu\_z\ \partial_z Alpha \\
& + 0.3333333333333333\ Alpha\ \ (2.0\ theta + trK)^2 \\
& + Alpha\ kappa\_z1\ \ (1.0 - kappa\_z2)\ \ theta
\end{aligned}
\tag{141}
$$

$$
\begin{aligned}
backward(x,y,z,t) = & + \min\{0, Betau\_x\}\ \partial_x trK \\
& + \min\{0, Betau\_y\}\ \partial_y trK + \min\{0, Betau\_z\}\ \partial_z trK
\end{aligned}
\tag{142}
$$

$$
\begin{aligned}
forward(x,y,z,t) = & + \max\{0, Betau\_x\}\ \partial_x trK \\
& + \max\{0, Betau\_y\}\ \partial_y trK + \max\{0, Betau\_z\}\ \partial_z trK
\end{aligned}
\tag{143}
$$

Evolution equation

$$
\partial_t theta = S^{(theta)} + Op(x,y,z,t) + forward(x,y,z,t) + backward(x,y,z,t) \tag{144}
$$

$$
\begin{aligned}
Op(x,y,z,t) = +0.5\ Alpha\ \ (&Rscalar + 0.6666666666666667\ trK^2 \\
&+ 0.6666666666666667\ theta\ \ ((-2.0\ theta) + trK) \\
&+ (-Atd\_xx\ Atu\_xx) + (-2\ Atd\_xy\ Atu\_xy) + (-2\ Atd\_xz\ Atu\_xz) \\
&+ (-Atd\_yy\ Atu\_yy) + (-2\ Atd\_yz\ Atu\_yz) + (-Atd\_zz\ Atu\_zz)) \\
&- Zu\_x\ \partial_x Alpha - Zu\_y\ \partial_y Alpha \\
&- Zu\_z\ \partial_z Alpha + (-8)\ \ Alpha\ \pi\ rho\_ADM \\
&- Alpha\ kappa\_z1\ \ (2.0 + kappa\_z2)\ \ theta
\end{aligned}
\tag{145}
$$

$$
\begin{aligned}
forward(x,y,z,t) = &+ \max\{0, Betau\_x\}\ \partial_x theta \\
&+ \max\{0, Betau\_y\}\ \partial_y theta + \max\{0, Betau\_z\}\ \partial_z theta
\end{aligned}
\tag{146}
$$

$$
\begin{aligned}
backward(x,y,z,t) = &+ \min\{0, Betau\_x\}\ \partial_x theta \\
&+ \min\{0, Betau\_y\}\ \partial_y theta + \min\{0, Betau\_z\}\ \partial_z theta
\end{aligned}
\tag{147}
$$

Auxiliary Field Equations

$$t\_optdepthe = optdepthe \tag{148}$$

$$t\_optdeptha = optdeptha \tag{149}$$

$$t\_optdepthx = optdepthx \tag{150}$$

$$t\_chie = chie \tag{151}$$

$$t\_chia = chia \tag{152}$$

$$t\_chix = chix \tag{153}$$

Construction
ρf, Yef, vfd_x, vfd_y, vfd_z, $epsf$, pf, $sqcs$, Tf, $dpfdrho$, $dpfdeps$, $dpfdye$, Efu_x, Efu_y, Efu_z
Algorithm
import
rule: c88048e2-2ca0-3cb5-835d-6acba9776fe1

name: con2prim

end import
$Alpha = |Alpha|$

$\rho f\_tmp = \rho f$

$$Yef\_tmp = \frac{DYf}{Df}$$

if $(Yef\_tmp < vacuum\_ye)$ **then**
$DYf = vacuum\_ye\_reset\ Df$

$Yef\_tmp = vacuum\_ye\_reset$
end if
if $(Yef\_tmp > ye\_maximum)$ **then**
$DYf = 0.9\ ye\_maximum\ Df$

$Yef\_tmp = 0.9\ ye\_maximum$
end if
$Tf\_tmp = Tf$

$pf\_tmp = pf$

$epsf\_tmp = epsf$

$vfd\_x\_tmp = vfd\_x$

$vfd\_y\_tmp = vfd\_y$

$vfd\_z\_tmp = vfd\_z$

$sqcs\_tmp = sqcs$

$dpfdrho\_tmp = dpfdrho$

$dpfdeps\_tmp = dpfdeps$

$dpfdye\_tmp = dpfdye$

$Sfd\_x\_tmp = Sfd\_x$

$Sfd\_y\_tmp = Sfd\_y$

$Sfd\_z\_tmp = Sfd\_z$

$tauf\_tmp = tauf$

$Df\_tmp = Df$

$DYf\_tmp = DYf$

$Bfu\_x\_tmp = Bfu\_x$

$Bfu\_y\_tmp = Bfu\_y$

$Bfu\_z\_tmp = Bfu\_z$

if $(externalCon2Prim = 1)$ **then**

$Efu\_x\_tmp = Efu\_x$

$Efu\_y\_tmp = Efu\_y$

$Efu\_z\_tmp = Efu\_z$

$externalCon2prim(Efu\_x\_tmp, Efu\_y\_tmp, Efu\_z\_tmp, rhof\_tmp, pf\_tmp, epsf\_tmp, vfd\_x\_tmp, vfd\_y\_tmp, vfd\_z\_tmp, sqcs\_tmp, dpfdrho\_tmp, dpfdeps\_tmp, Df\_tmp, Sfd\_x\_tmp, Sfd\_y\_tmp, Sfd\_z\_tmp, tauf\_tmp, W, chi, gtu\_xx, gtu\_xy, gtu\_xz, gtu\_yy, gtu\_yz, gtu\_zz, gtd\_xx, gtd\_xy, gtd\_xz, gtd\_yy, gtd\_yz, gtd\_zz, Bfu\_x\_tmp, Bfu\_y\_tmp, Bfu\_z\_tmp)$

$Efu\_x = Efu\_x\_tmp$

$Efu\_y = Efu\_y\_tmp$

$Efu\_z = Efu\_z\_tmp$

else

$vacuum\_D\_tmp = vacuum\_D\ decay\_factor$

$vacuum\_tau\_tmp = vacuum\_tau\ decay\_factor$

$vacuum\_rho\_reset\_tmp = vacuum\_rho\_reset\ decay\_factor$

$con2prim(eos\_type, \rho f\_tmp, Yef\_tmp, Tf\_tmp,$
$pf\_tmp, epsf\_tmp, vfd\_x\_tmp, vfd\_y\_tmp, vfd\_z\_tmp,$
$sqcs\_tmp, dpfdrho\_tmp, dpfdeps\_tmp, dpfdye\_tmp, \rho\_0,$
$\rho\_1, \rho\_2, a\_0, a\_1, a\_2, a\_3, K\_0, K\_1, K\_2, K\_3, gamma\_0,$
$gamma\_1, gamma\_2, gamma\_3, D, vacuum\_D\_tmp, Df\_tmp,$
$DYf\_tmp, sdetg, Sd\_x, Sd\_y, Sd\_z, Su\_x, Su\_y, Su\_z,$
$Sfd\_x\_tmp, Sfd\_y\_tmp, Sfd\_z\_tmp, tau, vacuum\_tau\_tmp,$
$tauf\_tmp, W, h, chi, gtu\_xx, gtu\_xy, gtu\_xz, gtu\_yy, gtu\_yz,$
$gtu\_zz, gtd\_xx, gtd\_xy, gtd\_xz, gtd\_yy, gtd\_yz, gtd\_zz, Bd\_x,$
$Bd\_y, Bd\_z, Bu\_x, Bu\_y, Bu\_z, Bfu\_x\_tmp, Bfu\_y\_tmp,$
$Bfu\_z\_tmp, max\_error, \gamma, inv\_chi, threshold\_vacuum,$
$threshold\_sqSmax, threshold\_sqBmax, energyShift,$
$vacuum\_ye\_beta, vacuum\_rho, vacuum\_rho\_reset\_tmp,$
$vacuum\_P\_reset, vacuum\_ye\_reset, vacuum\_temp\_reset,$
$vacuum\_tau\_reset, minTableTemperature, minTableEnergy)$

$$Efu\_x = \left(-\frac{(-vfd\_z\ Bfd\_y) + vfd\_y\ Bfd\_z}{sdetg^2} \right)$$
$$Efu\_y = \left(-\frac{vfd\_z\ Bfd\_x - vfd\_x\ Bfd\_z}{sdetg^2} \right)$$
$$Efu\_z = \left(-\frac{(-vfd\_y\ Bfd\_x) + vfd\_x\ Bfd\_y}{sdetg^2} \right)$$

end if

$\rho f = \rho f\_tmp$

$Yef = Yef\_tmp$

$Tf = Tf\_tmp$

$pf = pf\_tmp$

$epsf = epsf\_tmp$

$vfd\_x = vfd\_x\_tmp$

$vfd\_y = vfd\_y\_tmp$

$vfd\_z = vfd\_z\_tmp$

$sqcs = sqcs\_tmp$

$dpfdye = dpfdye\_tmp$

$dpfdeps = dpfdeps\_tmp$

$dpfdrho = dpfdrho\_tmp$

$Sfd\_x = Sfd\_x\_tmp$

$Sfd\_y = Sfd\_y\_tmp$

$Sfd\_z = Sfd\_z\_tmp$

$tauf = tauf\_tmp$

$Df = Df\_tmp$

$DYf = DYf\_tmp$

$Bfu\_x = Bfu\_x\_tmp$

$Bfu\_y = Bfu\_y\_tmp$

$Bfu\_z = Bfu\_z\_tmp$

optdepthe, *optdeptha*, *optdepthx*, *chie*, *chia*, *chix*, *qnu*, *rnu*
Algorithm
import
rule: e97e2d5a-04e5-3613-8e26-37de5c6d9379

name: leakage

end import
$optdepthe\_tmp = optdepthe$

$optdeptha\_tmp = optdeptha$

$optdepthx\_tmp = optdepthx$

$chie\_tmp = chie$

$chia\_tmp = chia$

$$chix\_tmp = chix$$
$$qnu\_tmp = qnu$$
$$rnu\_tmp = rnu$$
$$qnu\_e\_tmp = qnu\_e$$
$$qnu\_a\_tmp = qnu\_a$$
$$qnu\_x\_tmp = qnu\_x$$
$$betabeta = (gtd\_xx\ Betau\_x\ Betau\_x + gtd\_yy\ Betau\_y\ Betau\_y + gtd\_zz\ Betau\_z\ Betau\_z + 2\ gtd\_xy\ Betau\_x\ Betau\_y + 2\ gtd\_xz\ Betau\_x\ Betau\_z + 2\ gtd\_yz\ Betau\_y\ Betau\_z)$$
$$betav = (gtd\_xx\ Betau\_x\ vfd\_x + gtd\_yy\ Betau\_y\ vfd\_y + gtd\_zz\ Betau\_z\ vfd\_z + gtd\_xy\ Betau\_x\ vfd\_y + gtd\_xy\ Betau\_y\ vfd\_x + gtd\_xz\ Betau\_x\ vfd\_z + gtd\_xz\ Betau\_z\ vfd\_x + gtd\_yz\ Betau\_y\ vfd\_z + gtd\_yz\ Betau\_z\ vfd\_y)$$
$$velr = (vfd\_x\ x + vfd\_y\ y + vfd\_z\ z)$$
$$velr = \frac{velr}{\sqrt{x^2 + y^2 + z^2 + 0.01}}$$
$$f\_redshift = sqW\ \ (Alpha - betav)\ \ (1 + velr)$$
$$f\_redshift = f\_redshift\ \sqrt{Alpha^2 - betabeta}$$
$$leakage(do\_leakage, vacuum\_rho, threshold\_leakage\_vacuum, energyShift, optdepthe\_tmp, optdeptha\_tmp, optdepthx\_tmp, qnu\_e\_tmp, qnu\_a\_tmp, qnu\_x\_tmp, chie\_tmp, chia\_tmp, chix\_tmp, qnu\_tmp, rnu\_tmp, \rho f, epsf, Yef, Tf, sdetg, optdepthch, optdepthah, optdepthxh, optdepthev, optdepthav, optdepthxv, optdepthed, optdepthad, optdepthxd, chich, chiah, chihh, chiev, chiav, chixv, chied, chiad, chixd, f\_redshift)$$

$$TauN\_x = Op(x, y, z, t) \tag{154}$$

$$Op(x, y, z, t) = -faceta\ C\_N\ HN\_x \tag{155}$$

$$chie = chie\_tmp$$

$$TauN\_y = Op(x, y, z, t) \tag{156}$$

$$Op(x, y, z, t) = -faceta\ C\_N\ HN\_y \tag{157}$$

$$qnu\_x = qnu\_x\_tmp$$
$$qnu = qnu\_tmp$$
$$rnu = rnu\_tmp$$

$$TauN\_z = Op(x, y, z, t) \tag{158}$$

$$Op(x, y, z, t) = -faceta\ C\_N\ HN\_z \tag{159}$$

$$TauNe\_x = Op(x, y, z, t) \tag{160}$$

$$Op(x, y, z, t) = -faceta\ C\_N\ HNe\_x \tag{161}$$

$$TauNe\_y = Op(x, y, z, t) \tag{162}$$

$$Op(x, y, z, t) = -faceta\ C\_N\ HNe\_y \tag{163}$$

$$TauNe\_z = Op(x, y, z, t) \tag{164}$$

$$Op(x, y, z, t) = -faceta\ C\_N\ HNe\_z \tag{165}$$

$$TauM\_xy = Op(x, y, z, t) \tag{166}$$

$$Op(x, y, z, t) = -faceta\ C\_M\ HM\_xy \tag{167}$$

$$TauM\_xz = Op(x, y, z, t) \tag{168}$$

$$Op(x, y, z, t) = -faceta\ C\_M\ HM\_xz \tag{169}$$

$$TauM\_yz = Op(x, y, z, t) \tag{170}$$

$$Op(x, y, z, t) = -faceta\ C\_M\ HM\_yz \tag{171}$$

$$TauT\_xx = Op(x, y, z, t) \tag{172}$$

$$Op(x, y, z, t) = -faceta\ C\_T\ HT\_xx \tag{173}$$

$$TauT\_xy = Op(x, y, z, t) \tag{174}$$

$$Op(x, y, z, t) = -faceta\ C\_T\ HT\_xy \tag{175}$$

$$TauT\_xz = Op(x, y, z, t) \tag{176}$$

$$Op(x, y, z, t) = -faceta\ C\_T\ HT\_xz \tag{177}$$

$$TauT\_yy = Op(x, y, z, t) \tag{178}$$

$$Op(x, y, z, t) = -faceta\ C\_T\ HT\_yy \tag{179}$$

$$TauT\_yz = Op(x, y, z, t) \tag{180}$$

$$Op(x, y, z, t) = -faceta\ C\_T\ HT\_yz \tag{181}$$

$$TauT\_zz = Op(x, y, z, t) \tag{182}$$

$$Op(x, y, z, t) = -faceta\ C\_T\ HT\_zz \tag{183}$$

Auxiliary Variable Equations

Auxiliary variable equation

$$Bvf = \tag{184}$$

Auxiliary variable equation

$$optdepth eh = \tag{185}$$

Auxiliary variable equation

$$optdepth ev = \tag{186}$$

Auxiliary variable equation

$$optdepth ed = \tag{187}$$

Auxiliary variable equation

$$optdepth ah = \tag{188}$$

Auxiliary variable equation

$$optdepth av = \tag{189}$$

Auxiliary variable equation

$$optdepth ad = \tag{190}$$

Auxiliary variable equation

$$optdepth xh = \tag{191}$$

Auxiliary variable equation

$$optdepth xv = \tag{192}$$

Auxiliary variable equation

$$optdepthxd = \tag{193}$$

Auxiliary variable equation

$$chieh = \tag{194}$$

Auxiliary variable equation

$$chiev = \tag{195}$$

Auxiliary variable equation

$$chied = \tag{196}$$

Auxiliary variable equation

$$chiah = \tag{197}$$

Auxiliary variable equation

$$chiav = \tag{198}$$

Auxiliary variable equation

$$chiad = \tag{199}$$

Auxiliary variable equation

$$chixh = \tag{200}$$

Auxiliary variable equation

$$chixv = \tag{201}$$

Auxiliary variable equation

$$chixd = \tag{202}$$

Auxiliary variable equation

$$sqbc = \tag{203}$$

Auxiliary variable equation

$$sqca = \tag{204}$$

Auxiliary variable equation

$$sqa = \tag{205}$$

Auxiliary variable equation

$$W = \tag{206}$$

Auxiliary variable equation

$$h = \tag{207}$$

Auxiliary variable equation

$$D = \tag{208}$$

Auxiliary variable equation

$$tau = \tag{209}$$

Auxiliary variable equation

$$Sd\_x = \tag{210}$$

Auxiliary variable equation

$$Sd\_y = \tag{211}$$

Auxiliary variable equation

$$Sd\_z = \tag{212}$$

Auxiliary variable equation

$$Su\_x = \tag{213}$$

Auxiliary variable equation

$$Su\_y = \tag{214}$$

Auxiliary variable equation

$$Su\_z = \tag{215}$$

Auxiliary variable equation

$$Bd\_x = \tag{216}$$

Auxiliary variable equation

$$Bd\_y = \tag{217}$$

Auxiliary variable equation

$$Bd\_z = \tag{218}$$

Auxiliary variable equation

$$Bu\_x = \tag{219}$$

Auxiliary variable equation

$$Bu\_y = \tag{220}$$

Auxiliary variable equation

$$Bu\_z = \tag{221}$$

Auxiliary variable equation

$$faceta = \tag{222}$$

Auxiliary variable equation

$$kappa\_cc = \tag{223}$$

Auxiliary variable equation

$$kappa\_z1 = \tag{224}$$

Auxiliary variable equation

$$kappa\_z2 = \tag{225}$$

Auxiliary variable equation

$$feta = \tag{226}$$

Auxiliary variable equation

$$chi\_max = \tag{227}$$

Auxiliary variable equation

$$inv\_chi = \tag{228}$$

Auxiliary variable equation

$$detgtd = \tag{229}$$

Auxiliary variable equation

$$idetgtd = \tag{230}$$

Auxiliary variable equation

$$gtu\_xx = \tag{231}$$

Auxiliary variable equation

$$gtu\_xy = \tag{232}$$

Auxiliary variable equation

$$gtu\_xz = \tag{233}$$

Auxiliary variable equation

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