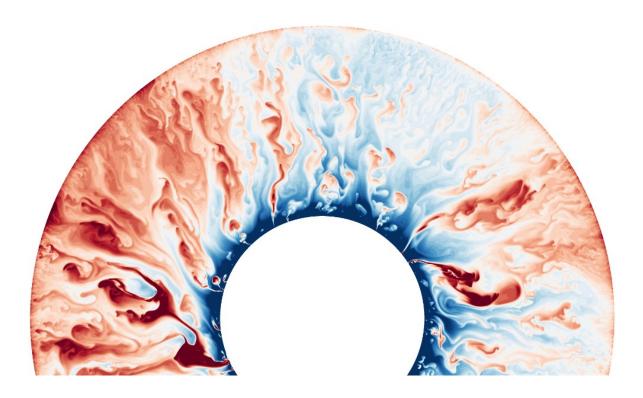
XSHELLS 2.8 : User Manual

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Chapter 1

Getting started

1.1 Description

XSHELLS is yet another code simulating incompressible fluids in a spherical cavity. In addition to the Navier-Stokes equation with an optional Coriolis force, it can also time-step the coupled induction equation for MHD (with imposed magnetic field or in a dynamo regime), as well as the temperature (and concentration) equation in the Boussinesq framework.

XSHELLS uses finite differences (second order) in the radial direction and spherical harmonic decomposition (pseudo-spectral). The time-stepping uses a highly stable, second order, semi-implicit, predictor-corrector scheme (with only diffusive terms treated implicitly).

XSHELLS is written in C++ and designed for speed. It uses the blazingly fast spherical harmonic transform library SHTns, as well as hybrid parallelization using OpenMP and/or MPI. This allows it to run efficiently on your laptop or on parallel supercomputers. A post-processing program is provided to extract useful data and export fields to python/matplotlib or paraview.

XSHELLS is free software, distributed under the CeCILL Licence (compatible with GNU GPL): everybody is free to use, modify and contribute to the code.

1.2 Easy installation

Grab the XSHELLS archive and extract it. Alternativley, you can use git to clone the repository

git clone --recurse-submodules https://bitbucket.org/nschaeff/xshells.git

Since v2.8, an interactive installation wizard is provided. Simply run:

./setup

and follow the instructions. You should soon be ready to setup configuration files, see 1.4.

1.3 Manual installation

If you need to install XSHELLS manually (e.g. because the above easy installation fails or you need a specific configuration not covered by the wizard), here are some useful details.

1.3.1 Requirements

The following items are *required*:

- a Unix like system (like linux),
- a C++ compiler,
- the FFTW library, or the intel MKL library.
- the SHTns library, which is included as a git submodule.

The following items are *recommended*, but not mandatory:

- a C++ compiler with OpenMP 4 support (for task dependencies),
- an MPI library (with thread support),
- the HDF5 library for post-processing and interfacing with paraview,
- Python 3, with NumPy and matplotlib (Python 2.7 should also work),
- Gnuplot, for real-time plotting.
- EVTK, for converting slices to VTK format for paraview.

1.3.2 Installing FFTW

FFTW (or the intel MKL) library must be installed first. FFTW comes already installed on many systems, but in order to get high performance, you should install it yourself, and use the optimization options that correspond to your machine (e.g. --enable-avx). Please refer to the FFTW installation guide.

Note that it is possible to use the intel MKL library instead of FFTW. To do so, you must configure XSHELLS (and SHTns if you configure it separately) with the --enable-mkl option.

1.3.3 Configuring XSHELLS

Get the XSHELLS archive and extract it. Alternativley, you can use git to clone the repository

```
git clone --recurse-submodules https://bitbucket.org/nschaeff/xshells.git
```

From the XSHELLS source directory, run:

./configure

to automatically configure XSHELLS for your machine. This will also automatically configure SHTns and compile it. Type ./configure --help to see available options, among which --disable-openmp and --disable-mpi, but also --enable-mkl to use the intel MKL library instead of FFTW.

Before compiling, you need to setup the xshells.hpp configuration file (see next section). To check that everything works, you may also want to run the tests, with python test.py or equivalently make test.

1.4 Configuration files

There are two configuration files:

- xshells.hpp is read by the compiler (compile-time options), and modifying it requires to recompile the program. The corresponding options are detailed in section 2.2.
- xshells.par is read by the program at startup (runtime options) and modifying it does not require to recompile the program. This file is detailed in section 2.1.

See chapter 2 for more details. Example configuration files can be found in the **problems** directory.

Before compiling, copy the configuration files that correspond most closely to your problem. For example, the geodynamo benchmark:

```
cp problems/geodynamo/xshells.par .
cp problems/geodynamo/xshells.hpp .
```

and then edit them to adjust the parameters (see sections 2.2 and 2.1).

1.4.1 Transition from versions 1.x to 2.x

Due to the change of time-stepping scheme, in xshells.par:

- adjust time-steps (x3 to x4)
- adjust constants for variable time-step

Due to internal changes, in xshells.hpp:

• some variables have their name changed. see examples.

Slices written by **xspp** are now in Numpy format (instead of text format). As a consequence, matlab/octave display scripts have been dropped.

1.5 Compiling and Running

When you have properly set the **xshells.hpp** and **xshells.par** files, you can compile and run in different flavours:

Parallel execution using OpenMP with as many threads as processors (recommended for laptops or for running on small servers):

```
make xsbig
./xsbig
```

Parallel execution using OpenMP with (e.g.) 4 threads:

```
make xsbig
OMP_NUM_THREADS=4 ./xsbig
```

Parallel execution using MPI with (e.g.) 4 processes:

```
make xsbig_mpi
mpirun -n 4 ./xsbig_mpi
```

Parallel execution using OpenMP and MPI simultaneously (hybrid parallelization), with (e.g.) 2 processes and 4 threads per process:

```
make xsbig_hyb
OMP_NUM_THREADS=4 mpirun -n 2 ./xsbig_hyb
```

Parallel execution using MPI in the radial direction and OpenMP in the angular direction, with (e.g.) 16 processes and 8 threads per process:

make xsbig_hyb2
OMP_NUM_THREADS=8 mpirun -n 16 ./xsbig_hyb2

Note that xsbig_hyb2 requires the OpenMP-enabled SHTns (./configure --enable-openmp)

Batch schedulers

Some examples for various batch schedulers and super-computers are also available in the batch folder.

1.6 Outputs

All output files are suffixed by the job name as file extension, denoted job in the following. The various output files are:

- xshells.par.job : a copy of the input parameter file xshells.par, for future reference.
- xshells.hpp.job : a stripped-out version of the file xshells.hpp that was used during compilation, for future reference.
- energy.job : a record of energies and other custom diagnostics. Each line of this text file is an iteration.
- fieldX0.job : the imposed (constant) field X, if any.
- fieldX_####.job : the field X at iteration number ####, if parameter movie was set to a non-zero value in xshells.par.
- fieldXavg_####.job : the field X averaged between previous iteration and iteration number ####, if parameter movie was set to 2 in xshells.par.
- fieldX.job : the last full backup of field X, or field X at the end of the simulation. Used when restarting a job.

All field files are binary format files storing the spherical harmonic coefficients of the field. To produce plots and visualizations, they can be post-processed using the xspp program (see chapter 3).

1.7 Limitations and advice for parallel execution

The parallelization of XSHELLS is done by domain decomposition in the radial direction only, using MPI. In addition to this domain decomposition, shared memory parallelism is implemented using OpenMP. Furthermore, MPI-3 shared memory can be used for communications between processes on the same node, with significant performance gains, and somehow mimicking a two-dimensional domain decomposition. The MPI-3 shared memory solver in XSHELLS requires a cache-coherent architecture, and is currently enabled by default only on x86 machines. To enable or tune MPI-3 shared memory, add -shared_mem=N to the command line, with N the number of *cores* that should share memory.

There are four variants of the code that differ in the way OpenMP is used:

• xsbig uses only OpenMP in the radial direction (no MPI). It can only run on a single node, but does not need an MPI library. This is good for a general purpose desktop or laptop computer, but also on NUMA nodes (although some MPI may lead to better performance). Low memory usage.

- xsbig_mpi uses only MPI in the radial direction (no OpenMP). This is good for medium sized problems, running on small clusters.
- xsbig_hyb uses both MPI and OpenMP in the radial direction, to reduce the number of MPI processes and memory usage. As a consequence, it is more efficient to use a number of radial shells that is a multiple of the number of computing cores. Beyond 1 thread per radial shell (or in case of imbalance), OpenMP tasks are used to share the work of one shell across several threads, although this leads to a less than ideal scaling.
- xsbig_hyb2 uses MPI in the radial direction and OpenMP within a radial shell. This strategy usually performs well, allows to efficiently address more computing cores, but uses a lot of memory (an order of magnitude more than xsbig_hyb). It is highly recommended to use a number of radial shells that is a multiple of the number of nodes. This mode cannot go beyond 1 MPI process per radial shell, but intra-node shared memory parallelism allows to use all cores of a single node within each process. Note also that the SHTns library compiled with OpenMP is needed.

In any case, the number of MPI processes cannot exceed the total number of radial shells. It is often more efficient to use a small number of MPI processes per node (1 to 4) and use OpenMP to have a total number of threads equal to the number of cores.

Because there is no automatic load balancing, some situations where the same amount of work is not required for each radial shell will result in suboptimal scaling when the number of MPI processes is increased. Such situations include (i) solid conducting shells (e.g. a conducting inner core) and (ii) variable spherical harmonic degree truncation (e.g. in a full-sphere problem). In these cases, especially the latter, use pure OpenMP or minimize the number of MPI processes.

Using MPI executables (including hybrid MPI+OpenMP) is thus optimal only if the following conditions are both met:

- all fields span the same radial domain (no conducting solid shells);
- the radial domain does not include the center r = 0 (and XS_VAR_LTR is not used, see section 2.2.4).

In such cases, XSHELLS should scale very well up to the limit of 1 thread per radial shell, and even beyond with the in-shell OpenMP mode of xsbig_hyb2 (see scaling example in Figure 1.1).

Advice for various hardwares:

• Intel servers (Xeon): Best performance is usually obtained with 1 or 2 MPI processes per socket. Tuning the use of MPI-3 shared-memory across one socket or one node may be useful. Add -shared_mem=N to the command line, where N is the number of cores of each CPU (socket) or each node. Using shared-memory across a

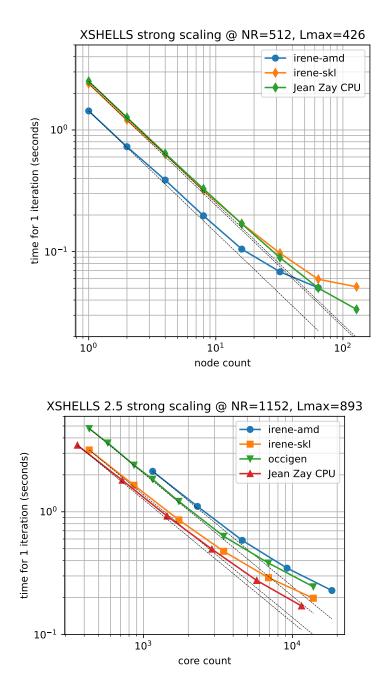


Figure 1.1: Performance scaling of XSHELLS, on french supercomputers with different architectures: AMD Epyc Rome on *irene-amd* (TGCC); Intel SkylakeX on *irene-skl* (TGCC); Intel Haswell on Occigen (CINES); and Intel Cascade Lake on Jean Zay CPU (IDRIS). The ideal scaling for each case is represented by the dashed black lines. Top: geodynamo simulation with $N_r = 512$ radial grid points and spherical harmonics truncated after degree $L_{max} = 426$. Bottom: geodynamo simulation with $N_r = 1152$ and $L_{max} = 893$.

whole node may help scaling to larger core counts, but can reduce raw performance (depending on machines).

- AMD Epyc 2nd generation (Rome): 4 OpenMP threads should give the best performance. Hence, one should use a number of MPI processes 4 times less the number of physical cores.
- AMD Epyc 3rd generation (Milan): 8 OpenMP threads (but 4 on some machines) should give the best performance. Hence, one should use a number of MPI processes 8 times (4 times) less the number of physical cores.

1.8 Citing

If you use XSHELLS for research work, you should cite the SHTns paper (because the high performance of XSHELLS is mostly due to the blazingly fast spherical harmonic transform provided by SHTns):

N. Schaeffer, *Efficient Spherical Harmonic Transforms aimed at pseudo-spectral numerical simulations*, Geochem. Geophys. Geosyst. **14**, 751-758, doi:10.1002/ggge.20071 (2013)

In addition, depending on the problem you solve, you could cite:

• Geodynamo:

N. Schaeffer et. al, *Turbulent geodynamo simulations: a leap towards Earth's core*, Geophys. J. Int. doi:10.1093/gji/ggx265 (2017)

• Spherical Couette:

A. Figueroa et. al, Modes and instabilities in magnetized spherical Couette flow, J. Fluid Mech. doi:10.1017/jfm.2012.551 (2013)

- Full-spheres: E. Kaplan et. al, Subcritical thermal convection of liquid metals in a rapidly rotating sphere, Phys. Res. Lett. doi:10.1103/PhysRevLett.119.094501 (2017)
- Kinematic dynamos:

N. Schaeffer et. al, Can Core Flows inferred from Geomagnetic Field Models explain the Earth's Dynamo ?, Geophys. J. Int. doi:10.1093/gji/ggv488 (2016)

Chapter 2

Setting up the simulation

Example configuration files can be found in the problems directory.

2.1 Run-time options: xshells.par

The file xshells.par is a simple text file. Each line may contain a single statement like var = expression, or a comment starting with #. A simple math parser allows to use convenient expressions like sqrt(4*pi/3).

All the following features can be set in xshells.par. There is no need to recompile if this file is changed, as it is read and interpreted at program startup.

2.1.1 Equations and controlling parameters

XSHELLS can time-step the Navier-Stokes equation in a rotating reference frame. Optionally it can include (i) a buoyancy force in the Boussinesq approximation, where the buoyancy has one or two sources obeying distinct advection-diffusion equations; and (ii) a Lorentz (or Laplace) force for conducting fluids where the magnetic field obeys the induction equation. Precisely, the following equations can be time-stepped by XSHELLS:

$$\partial_t \mathbf{u} + (2\mathbf{\Omega}_0 + \nabla \times \mathbf{u}) \times \mathbf{u} = -\nabla p^* + \nu \nabla^2 \mathbf{u} + (\nabla \times \mathbf{b}) \times \mathbf{b} + (c+T) \nabla \Phi_0$$
(2.1)

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b} - \eta \nabla \times \mathbf{b}) \tag{2.2}$$

$$\partial_t T + \mathbf{u} \cdot \nabla (T + T_0) = \kappa \nabla^2 T \tag{2.3}$$

$$\partial_t c + \mathbf{u} \cdot \nabla(c + C_0) = \kappa_c \nabla^2 c \tag{2.4}$$

$$\nabla \mathbf{u} = 0 \tag{2.5}$$

$$\nabla \mathbf{b} = 0 \tag{2.6}$$

where

• **u** is the velocity field.

- T and c are respectively the temperature and concentration fields, that contribute to the buoyancy in the Boussinesq formulation.
- $\sqrt{\mu_0 \rho} \mathbf{b}$ is the magnetic field (ρ and μ_0 are the fluid density and magnetic permeability, but are not relevant for this equation set).
- ν is the kinematic viscosity of the fluid and is set by the variable nu in xshells.par.
- $\eta = (\mu_0 \sigma)^{-1}$ is the magnetic diffusivity of the fluid (σ is its conductivity) and is set by the variable eta in xshells.par. Diffusivity $\eta(r)$ depending on shell radius are also supported (see sec. 2.2.3).
- κ and κ_c are respectively the thermal and chemical diffusivities of the fluid and are set by the variable kappa and kappa_c in xshells.par.
- Ω_0 is the rotation vector of the reference frame, which is usually along the vertical axis $\mathbf{e_z}$. It is set by the variable OmegaO in xshells.par, while the angle (in radians) between $\mathbf{e_z}$ and Ω_0 is set by OmegaO_angle (0 by default). Note that Ω_0 is always in the x z plane ($\phi = 0$).
- Φ_0 is the gravity potential (independent of time), controlled by field phi0 in xshells.par.
- T_0 and C_0 are the imposed base temperature and concentration profiles, controlled by fields tp0 and c0 in xshells.par.
- p^* is the dynamic pressure deviation (from hydrostatic equilibrium), which is eliminated by taking the curl of equation 2.1.

Equation 2.1 (respectively 2.2, 2.3 and 2.4) is time-stepped when u (respectively b, tp and c) is set to an initial condition in xshells.par. Disabling an equation is as easy as removing or commenting out the corresponding initial condition in xshells.par. Note that currently, the concentration equation cannot be time-stepped without the temperature equation.

```
Example If the following lines are found in xshells.par:

nu = 1.0

eta = sqrt(10)

OmegaO = 2*pi*1e3

u = 0

b = 0

#tp = 0

then the viscosity is set to \nu = 1.0, the magnetic diffusivity is set to

\eta = \sqrt{10}, and the rotation rate is set to \Omega_0 = 2\pi \times 10^3. The Navier-

Stokes (2.1) and the induction equation (2.2) will be time-stepped, but

not the temperature and concentration equations (2.3), simulating an

isothermal fluid.
```

Note that it is up to the user to choose dimensional or non-dimensional control parameters. A notable exception is the magnetic field, which is always scaled to the same unit as the velocity field.

MHD without Lorentz force (e.g. kinematic dynamos)

The Lorentz force can be turned off. Just add no_jxb = 1 in the xshells.par file.

Internal representation of vector fields

Vector fields are represented internally using a poloidal/toroidal decomposition:

$$\mathbf{u} = \nabla \times (T\mathbf{r}) + \nabla \times \nabla \times (P\mathbf{r}) \tag{2.7}$$

where \mathbf{r} is the radial position vector, and T and P are the toroidal and poloidal scalars respectively. This decomposition ensures that the vector field \mathbf{u} is divergence-free.

The scalar fields T and P for each radial shell are then decomposed on the basis of spherical harmonics.

2.1.2 Boundary conditions

Magnetic field boundary conditions are that of an electrical insulator outside the computation domain, with or without external sources of magnetic field (see section 2.1.3 for externally imposed magnetic fields).

Temperature or concentration boundary conditions can be fixed value (defined by the T_0 or C_0 profile), fixed flux (defined by $\partial_r T_0$ or $\partial_r C_0$), or Robin (such as $\partial_r T + \beta T = 0$). For Robin, the constant β (related to the Biot number) is set using Biot_Ti and Biot_To for the inner and outer boundaries respectively (or Biot_Ci and Biot_Co for the composition field).

Velocity boundary conditions can be zero, no-slip (with arbitrary prescribed velocity at the boundary) or stress-free.

The inner and outer boundary conditions can be chosen independently. The BC_U (for velocity), BC_T (for temperature) and BC_C (for concentration) entries in xshells.par allow to select the appropriate boundary conditions.

Example

The following lines in xshells.par define zero velocity and fixed temperature boundary condition at the inner boundary, and no-slip and fixed flux boundary condition at the outer boudnary.

2.1.3 Initial conditions and imposed fields

Predefined fields

Several predefined fields are defined in xshells_init.cpp. The command ./list_fields prints a list of these predefined fields, with their name in the first column. You can simply use this name in the xshells.par file to define an initial condition. You can also add your own by editing xshells_init.cpp.

Imposed fields are only supported for the gravity potential phi0 and for the basic state of temperature tp0 and concentration c0. Imposed magnetic fields can be obtained through the appropriate boundary conditions (magnetic fields generated by currents outside the computation domain only). Some predefined magnetic field include these boundary conditions, making them actually *imposed* fields (and are labeled as such). Note also that a linear mode exist which support arbitrary base fields (see §2.2.7).

Example The following lines in xshells.par set up the geodyanmo benchmark initial conditions.

```
E = 1e-3
Pm = 5
Ra = 100
u = 0
                      # initial velocity field
b = bench2001*5/sqrt(Pm*E)
                            # initial magnetic field (scaled by
                                 sqrt(1/(Pm*E)))
                            #
tp = bench2001*0.1
                      # initial temperature field
tp0 = delta*-1
                      # imposed (base) temperature field
phi0 = radial*Ra/E
                      # radial gravity field (multiplied by
                      # Ra/E to match geodynamo benchmark)
```

Field files as initial conditions

In addition, any field file can be given as initial condition. If the radial grid is not the same, the field must be interpolated on the new grid. To avoid mistakes, interpolation is disabled by default and must be enabled by interp = 1 (often found near the end of the xshells.par file).

Example The following lines start from the velocity field saved in file fieldU.previous_job, which was performed at different parameters and with a different number of radial grid points.

Sequential initialization of fields

It is sometimes convenient to combine several predefined fields, or read a file and overwrite with some predefined boundary conditions. Since v2.7, sequential initialization is supported. Simply separate the initial fields with a | operator. It is strongly advised to check that the sequential initialization produced the intended result, as some predefined fields overwrite everything while others only overwrite specified modes and/or shells. **Example** The following line start from the temperature field saved in file fieldT.previous_job, and enforces a new heterogeneous flux boundary condition read from file heatflux.txt. See also problems/heteroflux for an example where an heterogeneous flux is imposed.

```
tp = fieldT.previous_job | heteroflux(heatflux.txt)*0.1
```

2.1.4 Forcing

Besides thermal convection, mechanical forcing can be imposed at the boundaries.

Predefined variable a_forcing and w_forcing define the amplitude and frequency of a forcing. The precise nature of the forcing (e.g. differential rotation) must be defined in the xshells.hpp file before compilation (see section 2.2.6).

2.1.5 Spatial discretization

Radial grid

XSHELLS uses second order finite differences in radius. The total number of radial grid points is defined in xshells.par by the variable NR. The radial extent of each field is set using the corresponding R_X variable, which stores a pair of increasing positive real numbers defining the radial extent of the field. The NR grid points will be distributed between radii corresponding to the minimum and maximum of these values. Currently, only the magnetic field can extend beyond the velocity field, modeling conducting solid layers.

Example The following lines in **xshells.par** define the radial extent of the fields:

 $R_U = 7/13 : 20/13$ $R_B = 0.0 : 20/13$ $R_T = 7/13 : 20/13$

The default grid refines the number of points in the boundary layers, and this refinement can be controlled by the variable N_BL that stores a pair of integers, the first and second being the number of points reserved for the inner and outer boundary layer respectively, reinforcing the normal refinement. The code generating the grid can be found in the grid.cpp file.

Example The following lines in xshells.par define a grid with a total of 100 radial grid points, with 10 and 5 points reserved to the refinement of the inner and outer boundary layer respectively.

NR = 100 N_BL = 10,5

Alternatively, a radial grid can be loaded:

- from a text file containing the radius of each grid point (increasing) in a separate line.
- from a previously saved field (see section 1.6).

In both cases, simply indicate the filename in the r variable. It will override the NR and N_BL variables.

Example The following line in xshells.par will use the same grid as
the field stored in file fieldU_0001.previous
r = fieldU_0001.previous # load grid from file

Angular grid and spherical harmonic truncation

XSHELLS uses spherical harmonics to represent fields on the sphere:

$$f(\theta,\phi) = \sum_{m=0}^{M} \sum_{\ell=mK}^{L} f_{\ell}^{mK} Y_{\ell}^{mK}(\theta,\phi)$$
(2.8)

where Y_{ℓ}^m is the spherical harmonic of degree ℓ and order m. The expansion uses a K-fold symmetry in longitude (ϕ) and is truncated at maximum degree L and order MK. If K = 1 and M = L, it is the standard triangular truncation.

L, M and K are set in xshells.par using the Lmax, Mmax and Mres variables respectively. You must ensure that $L \ge MK$.

The angular grid (spanning the co-latitude θ and longitude ϕ) consists of Nphi regularly spaced points in longitude, and Nlat gauss nodes in latitude. If these are not specified, XSHELLS will choose the values for Nlat and Nphi in order to ensure best performance and no aliasing of modes (Nlat > 3L/2 and Nphi > 3M). **Example** These lines limit the spherical harmonic degree to 170. A 3-fold symmetry is used, and the maximum harmonic order is $56 \times 3 = 168$.

Lmax = 170 # max degree of spherical harmonics Mmax = 56 # max fourier mode (phi) Mres = 3 # phi-periodicity.

Most likely, 180 regularly spaced points in longitude and 256 gauss nodes in latitude will be used here.

2.1.6 Time-stepping

XSHELLS uses semi-implicit (or IMEX) time steppers, where the diffusive terms are treated implicitly, while all other terms (including non-linear terms) are treated explicitly. Since v2.4, XSHELLS offers a selection of time steppers:

- PC2 (the default) is a second order predictor-corrector scheme. Although this scheme requires three times more work per time-step than the classical Crank-Nicolson-Adams-Bashforth (CNAB) scheme, it allows time-steps from three to four times that of CNAB, leading to shorter time-to-solution¹.
- CNAB2 is the classical second order Crank-Nicolson-Adams-Bashforth (CNAB) scheme².
- SBDF2 is the semi-implicit Backward Difference Formula of order 2. This scheme³ has a stringent time-step restriction with the Coriolis force.
- SBDF3 is the semi-implicit Backward Difference Formula of third order⁴.
- BPR353 is a third order Runge-Kutta scheme featuring good stability and high accuracy⁵.

Simply add stepper = SBDF3 in the xshells.par file to use the SBDF3 scheme. Note that the allowable time steps can vary largely between these schemes. The automatic time-step selection (see below) should handle most problems well.

Both PC2 and CNAB2 use the Crank-Nicolson scheme for the implicit part, and as a consequence are not very accurate when computing decay rates (e.g. magnetic dipole decay rate). On the other hand SBDF2 and SBDF3 are accurate for decay rates, but non-linear terms like advection impose small time-steps for the scheme to remain stable.

²CNAB2: see eq. 2.9 of Wang & Ruuth (2008) https://www.jstor.org/stable/43693484.

¹before version 2.0, a classical CNAB scheme was used

³SBDF2: see eq. 2.8 of Wang & Ruuth (2008) https://www.jstor.org/stable/43693484.

⁴SBDF3: see eq. 2.14 of Wang & Ruuth (2008) https://www.jstor.org/stable/43693484.

⁵BPR353: see page A49 of Boscarino, Pareschi, Russo (2013) https://doi.org/10.1137/110842855 https://arxiv.org/abs/1110.4375.

The time-step of the numerical integration is set by dt in xshells.par.

The sub_iter variable is half the number of time-steps taken before any diagnostic is computed and written to file energy.job or displayed on screen. For example, if sub_iter = 50, then 100 time-steps will be performed before computing and printing some diagnostics. This is then called an iteration.

The iter_max variable is the total number of iterations, so that the total number of time-steps before the code will stop is iter_max $\times 2 \times \text{sub_iter}$.

By setting dt_adjust = 1, an automatic time-step adjustment can be turned on. The time-step adjustment can be controlled through advanced options (see §2.1.10). In that case, the number of sub-iterations sub_iter is also adjusted so that an iteration is a constant time span, and thus the outputs happen at fixed time intervals $\Delta T = 2 \times \text{sub_iter} \times \text{dt}$.

Finally, iter_save controls the number of iterations before a (partial) snapshot is saved to disk.

Example The following lines in xshells.par will use a time-step of 0.01 for the numerical integration:

tion). The program will stop after $iter_max=300$ outputs (or iterations), spanning a total physical time of $t_{end} - t_{start} = 150.0$. Partial fields are saved every 10 iterations, or every 5.0 physical time units, if movie = 1 is set (see below).

2.1.7 Real time plotting

At each iteration, XSHELLS can plot the kinetic and magnetic energies as a function of time, using gnuplot. Note that the plots are refreshed every iteration, but no more than once every two seconds. This allows to follow program execution in real-time, but might not be useful for high performance distributed jobs. The interaction with gnuplot must be turned on by passing the --enable-gnuplot option to ./configure.

The variable plot in the file xshells.par then allows some control:

• plot = 0: disables plotting.

- plot = 1: shows plot on display; if no display found, write to png file instead. This is the default.
- plot = 2: saves plot to png file only.
- plot = 3: shows plot on display (if available) and also saves plot to png file.

2.1.8 Time lapse field snapshots

The parameter **movie** controls the field snapshots, saved every **iter_save** iterations (see above).

- movie = 1: the initial field is saved to fieldX_0000.job, after iter_save iterations the fields are saved to fieldX_0001.job, then fieldX_0002.job, and so on.
- movie = 0 : no such fields are saved.
- movie = 2: in addition to the snapshots of the fields, the time-average since the last snapshot is also computed and saved.

The parameter prec_out controls the precision (single or double precision) of the snapshot files. The snapshots are saved in double precision by default (prec_out = 2), which allows restarting or computing gradients. If you need to save disk space, you can use single precision instead by setting set prec_out = 1, thus writing field files of half size. To save further disk space, snapshots can be truncated at lower spherical harmonic degree and order, using the parameters lmax_out and mmax_out, respectively.

Sometimes, single precision is not enough but double precision is not needed. Setting set prec_out = 3 uses the custom fp48 format which takes 25% less space than double precision, while keeping high accuracy.

The snapshots can then be post-processed with xspp to produce plots or movies (see 3). Note that field in fp48 format are read seamlessly by xspp which can also convert between formats. However, pyxshells cannot read the fp48 format.

Example The following lines in **xshells.par** instruct the program to output snapshots and time-averages of the axisymmetric component of the fields, every **iter_save** iterations:

2.1.9 Checkpointing and restarting capabilities

By default after initialization the job starts at the beginning (iteration 0). It is easy to start a new job by using as input fields the **field** files written by a previous job, effectively continuing that job.

Sometimes, it is useful to automatically continue a stopped or killed job (e.g. in batch execution environments found in high-performance computing machines). By default, a full resolution snapshot is written to disk every four hours. Parameters found in xshells.par allow to tune that interval, and enable restart from these checkpoint automatically when the program is run again.

For increased safety, when writing a new checkpoint (or backup) to file fieldX.jobname, the previous one is first renamed to fieldX_back.jobname. This file may allow to continue a simulation in case of an unexpected termination of the program while writing the new checkpoint.

If **restart** = 1, the program will start by looking in the current directory for checkpoint files that have been saved by a previous run with the same job name, and use these to resume that job.

Example

Suppose that on a supercomputer, the wall time of the jobs is limited to 24 hours. In order to run a job that spans several days, the following lines in xshells.par allow a job to be resumed by simply resubmitting it:

In addition, a checkpoint (or backup) is written to disk every 470 minutes, and the program will stop after writing the third backup, thus leaving 30 minutes of safety time for program initialization and file writing time. In case of a system failure, no more than 470 minutes of computing time will be lost.

2.1.10 Advanced options

Fine tuning of the automatic time-step selection is possible through some variables.

C_u is a safety factor for the standard CFL (based on the velocity and the grid size),

which depends on the time-scheme. For the PC2 time-scheme used by xshells, the default value is $C_u = 0.65$. You may need a more stringent value for a given problem, but if your simulation still does not work for a value lower than 0.1, the problem is probably elsewhere. C_alfv control the time-step adjustment (active if $dt_adjust=1$), regarding Alfvén criteria (due to magnetic field). The lower the values of C_u and C_alfv , the smaller the adjusted time step will be.

In addition, to prevent too many time step adjustments, if dt_tol_lo < dt/dt_target < dt_tol_hi, no time-step adjustment is done.

Example

$C_u = 0.65$	#	default:	0.65
C_cori = 1	#	default:	1
$C_{alfv} = 2.8$	#	default:	2.8
$dt_tol_1 = 0.9$	#	default:	0.9
dt_tol_hi = 1.05	#	default:	1.05

Variable spherical harmonic degree truncation is controlled by setting the variable rsat_ltr in xshells.par. This is possible only if XSHELLS was compiled with variable truncation enabled (see section 2.2.4).

Spectral convergence of fields is checked by comparing the maximum of the end of the spectrum (four last modes) with the maximum of the spectrum. The first few modes are ignored (because their amplitude can be related directly to forcing or imposed fields). The variables $sconv_lmin$ and $sconv_mmin$ that can be set in xshells.par define the first spherical harmonic degree and order respectively that are considered when checking spectral convergence in the ℓ or m spectra. The default value is 3 for both variables. Setting a value larger than Lmax disables spectral convergence checks. Note also that the convergence checks are skipped when only a few modes are computed.

In addition, the maximum allowed value for Sconv can be set using variable sconv_stop (default value is 0.3). When the computed Sconv is larger than this limit, the program stops. Setting sconv_stop = 0 disables this safety check.

Example

```
sconv_lmin = 3  # ignore l=0-2 for spectral convergence.
sconv_mmin = 1  # ignore m=0 for spectral convergence.
sconv_stop = 0.3  # set the maximum value of Sconv before the program stops.
```

The SHTns library can be controlled in terms of algorithm used for transforms, and in terms of polar optimization threshold.

The sht_type variable allows to constrain the transform method used:

- sht_type = 0 : select fastest method using a classic Gauss-Legendre grid (default setting).
- sht_type = 1 : select fastest method, allowing also regular grids (with DCT) which may be faster for small Mmax.
- sht_type = 2 : impose a regularly spaced grid (not recommended as it is often slower).
- sht_type = 3 : force a regularly spaced grid using DCT (not recommended as it is often slower).
- sht_type = 4 : debug mode; initialization time is reduced, but a default method is used (no selection of fastest method).
- sht_type = 6 : use a Gauss-Legendre grid with on-the-fly computation (preferred when parallel execution or big resolutions).

Finally, the polar optimization threshold can be adjusted with sht_polar_opt_max, the value below which coefficients near the pole are neglected. To give the reader some more insight, here are a some possible values and their impact:

- sht_polar_opt_max = 0 : no polar optimization.
- sht_polar_opt_max = 1e-14 : very safe optimization (default).
- sht_polar_opt_max = 1e-10 : safe optimization.
- sht_polar_opt_max = 1e-6 : aggressive optimization.

OpenMP tasks with dependencies are used by xsbig_hyb whenever the number of shells cannot be distributed evenly enough between the available threads. However, OpenMP tasks with dependencies can cause problems with some compilers (e.g intel compilers prior to version 18), and can be turned off by setting omp_tasks = -1, or forced with omp_tasks = 1 in the xshells.par file.

2.1.11 Beta features

The following features have not been thoroughly tested and may not work flawlessly in all situations. If you want to use them, please test and report bugs.

Non-linear terms in linear mode can be included to compute the saturation of an instability growing on a base flow. The program must be compiled in linear mode (see 2.2.7), and each non-linear term can be activated separately using a comma separated list in xshells.par:

```
nonlin = ugu,uxb,jxb,ugt,ugc
```

where ugu, uxb, jxb, ugt and ugc activate respectively $\mathbf{u} \cdot \nabla \mathbf{u}$, $\nabla \times (\mathbf{u} \times \mathbf{b})$, $(\nabla \times \mathbf{b}) \times \mathbf{b}$, $\mathbf{u} \nabla T$, $\mathbf{u} \nabla C$ for the perturbations $\mathbf{u}, \mathbf{b}, T$ and C in the equations (see 2.1.1).

Example To compute a kinematic dynamo growing on a saturated hydrodynamic instability, use:

```
nonlin = ugu,uxb  # include only u.grad(u) and curl(u x b)
```

2.2 Compile-time settings: xshells.hpp

All the following settings can be found in **xshells.hpp**. You have to recompile the program if you change this file.

2.2.1 Custom diagnostics

Enable by uncommenting:

#define XS_CUSTOM_DIAGNOSTICS

In addition to the total energy of the three fields U, B and T, which are saved every 2 sub_iter time steps (see section 2.1.6), custom diagnostics can be defined in the custom_diagnostic() function, found in the xshells.hpp file. They are computed every iteration and saved in energy.job after the energies. The best is to look at the existing diagnostics defined in the custom_diagnostic() function to add your own.

2.2.2 Variable time-step

Enable compilation of variable time-step code by uncommenting:

```
#define XS_ADJUST_DT
```

In addition, variable time-step must also be allowed by setting dt_adjust = 1 in file xshells.par (see also section 2.1.6).

2.2.3 Variable conductivity

In equation 2.2, conductivity can depend on radius r. To define a conductivity profile $\eta(r)$, uncomment:

#define XS_ETA_PROFILE

and then define your profile in the calc_eta() function, found in the xshells.hpp file. The purpose of calc_eta() is simply to fill the array etar with values of the magnetic diffusivity for every radial shell. The program handles continuous profiles as well as discontinuities in $\eta(r)$ properly and automatically.

2.2.4 Variable spherical harmonic degree truncation

In order to compute in a full sphere and avoid problems near r = 0, the spherical harmonic expansion must be truncated at low degree near r = 0. XSHELLS can truncate the spherical harmonic expansions at a different degree for each shell, when the following line is uncommented in xshells.hpp:

#define VAR_LTR 0.5

The value of VAR_LTR (0.5 in the line above, which is a good choice for full-sphere computations) is used as α in the formula to determine the truncation degree ℓ_{tr} :

$$\ell_{tr}(r) = L_{max} \sqrt{\frac{r}{\alpha r_{max}}} + 1$$

where L_{max} is defined by Lmax in xshells.par and r_{max} is the radius of the last shell. Note that ℓ_{tr} cannot exceed L_{max} .

Note also that α can be overridden by the rsat_ltr variable in xshells.par.

2.2.5 Hyper-diffusivities

To avoid computing small-scales that may not be dynamically relevant, the easy way is to employ hyper-diffusivity, which increase the diffusivity of small-scales. XSHELLS can be compiled with hyper-diffusivities when the following line is uncommented in xshells.hpp:

#define XS_HYPER_DIFF

The formulae used to compute the viscosity is the one proposed by Nataf & Schaeffer (2015):

$$\nu(\ell) = \begin{cases} \nu_0 & \text{for } l \le \ell_c \\ \nu_0 q^{\ell - \ell_c} & \text{for } \ell > \ell_c \end{cases}$$
(2.9)

with $q = (\nu_{max}/\nu_0)^{1/(L_{max}-\ell_c)}$. The cutoff ℓ_c and the ratio ν_{max}/ν_0 are set respectively by the hyper_diff_10 and hyper_nu parameters in xshells.par. If hyper_diff_10 = 0, hyper_diffusivity is disabled. Similarly, hyper_nu = 1 gives uniform viscosity, while hyper_nu = 100 leads to a viscosity at $\ell = L_{max}$ that is 100 times larger than at large scales.

Note that thermal and magnetic diffusivities can also be treated similarly, using respectively the hyper_kappa and hyper_eta parameters. The cutoff parameter ℓ_c is the same for all diffusivities.

2.2.6 Boundary forcing

Amplitude and frequency are set at runtime by a_forcing and w_forcing in the xshells.par file.

Time dependent boundary forcing are defined in the function calc_Uforcing(), found in the xshells.hpp file. In this function you must define a name for your forcing through the macro U_FORCING. The angular velocity of the solid bodies (defining the boundary of the fluid shell) can be set in this function. It will be used as a boundary condition for the flow if no-slip boundaries are used (see section 2.1.2).

See the example found in the problems/couette/ folder for more details, and uncomment the part of the function corresponding to your problem. In particular axial differential rotation of the inner or outer boundary can be used to simulate a spherical Couette flow; equatorial differential rotation (or spin-over) can be used to simulate precession or nutation.

Note that the rotation rate of the solid bodies is also used in the induction equation if the magnetic field extends into the solids (conducting solid shells).

Arbitrary stationary boundary flows You can impose arbitrary stationary flows at the solid boundaries. Uncomment:

#define XS_SET_BC

and change the set_U_bc() function found in xshells.hpp according to your needs. Note that the boundary conditions for the poloidal velocity field is stored in the shell beyond the first or last fluid shells (respectively NG-1 and NM+1). See for example the xshells.hpp file in the problems/full_sphere/ folder. Note that the solid should not be conducting if this feature is used, as no corresponding solid flow will be generated.

2.2.7 Linearized equations and base fields

To replace the equations with their linearized version (no $(\nabla \times \mathbf{u}) \times \mathbf{u}$, no $(\nabla \times \mathbf{b}) \times \mathbf{b}$, no $\mathbf{u} \cdot \nabla c$), uncomment:

#define XS_LINEAR

Note that spherical harmonic transforms are not needed anymore if there are no base fields, leading to a much faster program. Base fields are also supported, and can be set using u0, b0 and tp0 in the xshells.par file. See also the example located in problems/taw.

2.2.8 Mean-field dynamo

To include an axisymmetric alpha effect in the induction equation, add the following in the xshells.hpp file:

#define XS_MEAN_FIELD

And provide the alpha field using the init_alpha function. See the example located in problems/kinematic_dynamos.

Chapter 3

Post-processing and visualization

Several tools are available for post-processing and visualization of data produced by the XSHELLS code:

- xsplot.py: a python module for plotting 1D or 2D data.
- xsplot: command-line interface to xsplot.py.
- **xspp**: command-line tool for extracting slices, profiles, spectra and more from the field files.
- pyxshells.py: a python module that can load and handle the binary XSHELLS field files.

The python module xsplot is provided to load and display data from XSHELLS. It can be used interactively or within scripts. Such Python scripts using matplotlib and xsplot are located in the matplotlib dierectory, and can be called from command line.

xsplot can also be used directly from command line and it will guess the type of file and display it accordingly.

The python modules should be installed by calling make install-py, or explicitly python setup.py install --user from the python subdirectory. In addition, it is convenient to copy the small script python/xsplot in your path, so that you can invoke xsplot from any directory.

3.1 Plotting time-dependent quantities

Time-dependent quantities, such as the energies and other custom diagnostics which are stored in the energy.job file (see section 2.2.1) are also easy to plot with the xsplot tool.

Example By default, xsplot displays kinetic and magnetic energy as a
function of time:
xsplot energy.job
A list of all available quantities is also shown.
To plot other quantities (see section 2.2.1), for instance kinetic energy
and viscous dissipation, use:
xsplot energy.job -c Eu,D_nu

The load_diags function in the xsplot module can be used to retrieve conveniently any diagnostic. It returns a dictionary of time series for easy plotting.

Example To plot the magnetic to kinetic energy ratio as a function of time:

```
> from pylab import *
> import xsplot
> d = xsplot.load_diags('energy.job')
> t = d['t']
> plot(t, d['Eb']/d['Eu'])  # plot magnetic to kinetic energy ratio
```

3.2 Dealing with 3D fields

Fields are stored in binary files (see 1.6), using a custom format. They can be handled after the simulation by the xspp command line program. Alternatively, the pyxshells python module can also read, write and handle these files (see section 3.2.5).

3.2.1 Using the xspp command-line tool

Compile the program by typing make xspp. Invoking it without arguments (by running ./xspp) will print a help screen including the commands and their syntax.

Example The following will display information about the file fieldU.job (resolution, precision, time of the snapshot, ...):

./xspp fieldU.job

To compute the energy and maximum value of the curl of the field:

./xspp fieldU.job curl nrj max

To extract the field values along a line spanning the x-axis from x = -1 to x = 0.8, and also display total energy of field:

./xspp fieldU.job line -1,0,0 0.8,0,0 nrj

Add two fields and save the result to a new file (the first file will set the resolution for the result):

```
./xspp fieldT_0004.job + fieldT0.job save fieldT_total_0004.job
```

Extract only a given range of spherical harmonic coefficients (2 to 31) and computes the corresponding energy:

./xspp fieldB.job llim 2:31 nrj

Note that **xspp** is not parallelized using MPI, so that for very big cases you might run out of memory (although it can operate out-of-core – without actually loading the whole file in memory – in some cases). As a workaround you can always reduce the spherical harmonic truncation while reading your big files with the **llim** option (see example avobe).

3.2.2 Extract 2D slices

One of the most common usage for xspp is to extract two-dimensional slices of the 3D data stored in spectral representation in the field files. Four types of 2D slices are available:

- Meridian cuts (a plane containing the z-axis), with the merid command;
- Equatorial cuts (the plane z = 0), with the equat command;
- Surface data (on a sphere of given radius r), with the surf command;
- Disc cuts (an arbitrary plane), with the disc command;

When these commands are given to xspp, NumPy files corresponding to the required cuts are written to the current directory. These NumPy files (*.npy) can then be loaded and displayed using Python with NumPy and matplotlib (see next section).

Example A meridian cut at $\phi = 0$: ./xspp fieldU.job merid An equatorial cut, and a meridian cut at $\phi = 45$ degrees, of the vorticity (curl of U) ./xspp fieldU.job curl equat merid 45 Extract the field at the spherical surface closest to r = 0.9, using only the symmetric components. ./xspp fieldU.job sym 0 surf 0.9 Make a cut at z = 0.7, using 200 azimuthal points, with field truncated at harmonic degree 60: ./xspp fieldU.job llim 0:60 disc 200 0,0,0.7

3.2.3 plotting with python/matplotlib

The python module **xsplot** is provided to load and display cuts produced by **xspp**. It can be used interactively or within scripts. Such Python scripts using matplotlib and xsplot are located in the **matplotlib** dierectory, and can be called from command line. **xsplot** can also be used directly from command line and will guess the type of cut of your file and display it accordingly.

The python module should be installed by calling make install-py, or explicitly python setup.py install --user from the python subdirectory. In addition, it is convenient to copy the small script python/xsplot in your path, so that you can invoke xsplot from any directory.

Example Produce a meridian and an equatorial cut with **xspp**:

./xspp fieldU.job merid equat

From command prompt, quickly load and plot all components of the field in this meridional slice, as well as in the equatorial plane.

xsplot o_merid_0.npy o_equat.npy

xsplot has several convenient options. For instance you can plot the first two components only (using -c option), and specify a range for the colormap (using the -z option):

```
xsplot o_equat_0.npy -c 0,1 -z "(-1e-3,2e3)"
```

Alternatively, from an Ipython interpreter (or notebook, or script), load and plot the ϕ -component of the field in the meridional and equatorial slices:

```
> import xsplot
> a = xsplot.load_slice_npy('o_merid_0.npy')
> xsplot.plot_slice(a, 2)  # plot third (phi) component
> d = xsplot.load_slice_npy('o_equat.npy')
> xsplot.plot_slice(d, 1)  # plot second (phi) component
```

Several useful scripts for basic and advanced plotting can be found in the matplotlib directory.

3.2.4 3D visualization with paraview

From the paraview website: ParaView is an open-source, multi-platform data analysis and visualization application. ParaView users can quickly build visualizations to analyze their data using qualitative and quantitative techniques.

Full spatial fields can be saved to XDMF format, which can be loaded by paraview. Note that the HDF5 library is required for this to work, and must be found by the **configure** script. If so, Simply run:

```
make xspp
./xspp fieldB_0004.job hdf5 B_cartesian.h5
```

The file B_cartesian.h5.xdmf describes the cartesian components of the vector field B on a spherical grid that can be read directly by paraview (if prompted for a loader, select 'XDMF').

3.2.5 Advanced post-processing using pyxshells

For more complex post-processing, xspp may not be enough. The python module pyxshells allows you to quickly write your own scripts to work directly with the spectral fields stored in the field files output by XSHELLS, cast them to spatial domain, and so on.

To install the python modules, type make install-py. You can then import the pyxshells module from any python script. Note that the pyxshells module requires the shtns python module, which can be installed from the shtns directory using:

```
./configure --enable-python --disable-openmp
make
python setup.py install --user
```

Example Here is an example of what pyxshells can do:

```
> import pyxshells
> f = pyxshells.load_field('fieldU.job')
> r = f.grid.r  # the radial grid
> KE = f.energy()  # computes the energy
> f.sht.set_grid()  # prepare the spherical grid
> u3D = f.spat_full()  # the 3D field in spherical coordinates
> ur,ut,up = f.spat_shell(len(f.grid.r)-1)  # surface field
> ur,ut,up = f.spat_merid()  # meridional slice
> ur,ut,up = f.spat_equat()  # equatorial slice
```

Chapter 4

Hacking

4.1 Main source files

The main programs (xsbig_xsbig_mpi and xsbig_hyb) all share the same source code:

- **xshells_big.cpp** is the main source file, including the main loop, and all the solver logic.
- xshells.hpp is where a lot of customization goes on. See section 2.2.
- grid.cpp contains functions related to the radial grid and to the banded matrix linear solver.
- **xshells_spectral.cpp** contains the definition of the classes used to describe spectral fields (scalar and vector), and the implementation of most associated methods.
- **xshells_io.cpp** contains methods and functions to load and store fields to file on disk.
- **xshells_physics.cpp** generation of evolution matrices and computation of physical quantities.
- xshells_init.cpp initialization functions and predefined fields.
- findiff.cpp finite difference derivatives.
- xshells_linop.cpp Linear operators (banded matrices) and solvers.
- xshells_stepper.cpp Support of various time-steppers.
- xshells_sparse.cpp To handle the Coriolis force implicitely, we rely to large sparse matrices.

The post-processing program **xspp** uses the previous source files but also:

- xspp.cpp as main source file (instead of xshells_big.cpp)
- **xshells_spatial.cpp** contains the definition of the classes used to describe spatial fields (scalar and vector), their relationship with spectral representation and associated methods.
- **xshells_render.cpp** contains method implementations for rendering fields on grids and slices, as well as output to hdf5 files.

4.2 Doxygen

The source code also contains Doxygen documentation comments. Run make docs to generate the documentation targeted at developers and contributors in the doc/html/ folder.

4.3 Git repository

To track the changes to the code, the distributed version control system git is used. Two repositories exist.

- https://bitbucket.org/nschaeff/xshells provides the master and devel branches, and also a single package including the SHTns library for easy installation.
- https://gricad-gitlab.univ-grenoble-alpes.fr/schaeffn/xshells is targeted at developers, with more topic branches. In addition, it runs automatic tests.

Both allow to fork it and merge requests are welcome.

Chapter 5

Frequently Asked Questions

- Why is XSHELLS so fast? Short answer: it uses SHTns for spherical harmonic transforms and tries to preserve data locality. A Longer answer can be found in this presentation: http://dx.doi.org/10.6084/m9.figshare.1304532.
- What are the differences with the PARODY code? The numerical methods are basically the same, but their implementations are different. The PARODY code is written in Fortran. The performance and scalability of XSHELLS are better.
- Why is XSHELLS not written in Fortran? Because we don't like Fortran, and we would not be able to get the same level of performance out of a Fortran code. But maybe you could !