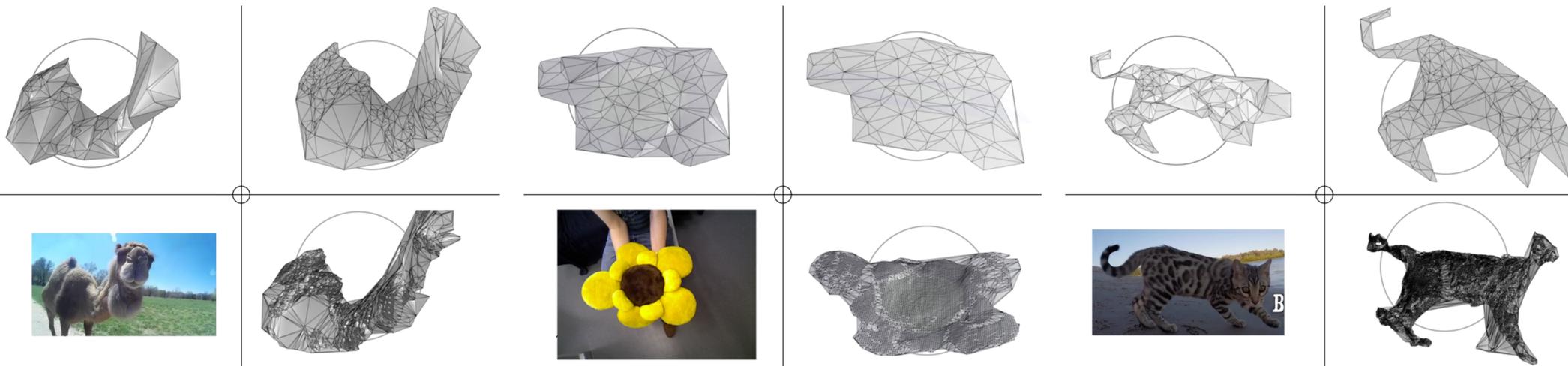


# Incremental Non-Rigid Structure-from-Motion with Unknown Focal Length

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Qualitative Results on YouTube videos. Comparison of our dense NRSfM method (bottom-right) to (Ji et al., 2017) (top-left) and (Dai et al., 2012) (top-right) on three different sequences.

## How to reconstruct deforming objects from your favorite YouTube video?

- Input: dense 2D point tracks (optical flow).
- Output: dense 3D non-rigid reconstruction of the sequence
- NRSfM techniques for dense reconstruction of deforming surfaces from a YouTube video?

YouTube video	SotA NRSfM methods
Unknown object	⇒ no Shape-from-Template (SfT)
Unknown camera	⇒ no camera calibration
Dense reconstruction	⇒ high computational complexity

## Isometric NRSfM

Reconstructing deforming objects from multiple images of a monocular camera is challenging.

- Infinite combinations of deformations and camera motions yield same projection
- ⇒ Severely underconstrained

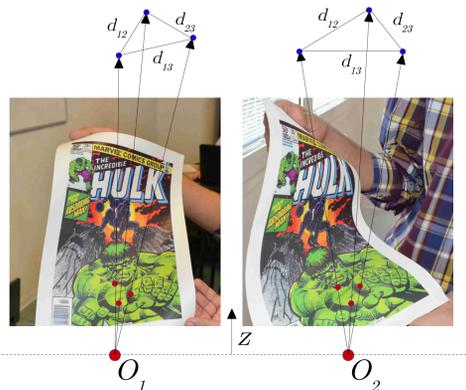
Isometric deformations.

- Geodesic distances are preserved
- Good approximation for many real deformations

## Original Formulation

Given intrinsics  $\mathbf{K}$ , template-less MDH can be written as a convex SOCP with inextensibility constraints (Chhatkuli et al., CVPR 2016):

$$\begin{aligned} \max_{\lambda_i, d_{ij}} \quad & \sum_l \sum_i \lambda_i^l, \\ \text{s.t.} \quad & \|\mathbf{K}^{-1}(\lambda_i^l \mathbf{u}_i^l - \lambda_j^l \mathbf{u}_j^l)\| \leq d_{ij}, j \in \mathcal{N}(i), \\ & \sum_i \sum_{j \in \mathcal{N}(i)} d_{ij} = 1. \end{aligned}$$



## Reconstruction Upgrade

Given a reconstruction under  $\hat{\mathbf{K}}$ , we can approximate the reconstruction under  $\mathbf{K}$  by upgrading the depth using

$$\lambda_i^l \approx \frac{\hat{\lambda}_i^l \|\hat{\mathbf{K}}^{-1} \mathbf{u}_i^l\|}{\|\mathbf{K}^{-1} \mathbf{u}_i^l\|}.$$

- Approximates local distances, error depends on point densities
- Relates reconstructions and distances as an analytic function of  $\mathbf{K}$
- No need for (expensive) re-reconstruction, given an initial reconstruction

## Non-Rigid Camera Calibration

Incorrect intrinsics  $\mathbf{K}$  affect angles between camera rays

- leads to distorted distances in reconstruction
- ⇒  $\mathbf{K}$  introduces errors in isometry  $\Phi(\mathbf{K})$ .
- ⇒ Isometry as certificate for correct intrinsics.

Based on this observation, we estimate the focal length in both template-based and template-less cases. The full intrinsics  $\mathbf{K}$  can then be recovered through non-linear refinement.

### Template-based

Isometric consistency w.r.t. template.

$$\Phi_T(\mathbf{K}) = \sum_i \sum_{j \in \mathcal{N}(i)} (d_{ij} - \hat{d}_{ij}(\mathbf{K}))^2.$$

Degree 2 polynomials on IAC can be derived to generate hypotheses.

### Template-less

Isometric consistency across views.

$$\Phi(\mathbf{K}) = \sum_k \sum_{l \neq k} \sum_i \sum_{j \in \mathcal{N}(i)} (\hat{d}_{ij}^k(\mathbf{K}) - \hat{d}_{ij}^l(\mathbf{K}))^2.$$

Optimize by focal length a sweeping algorithm with iterative re-reconstruction.

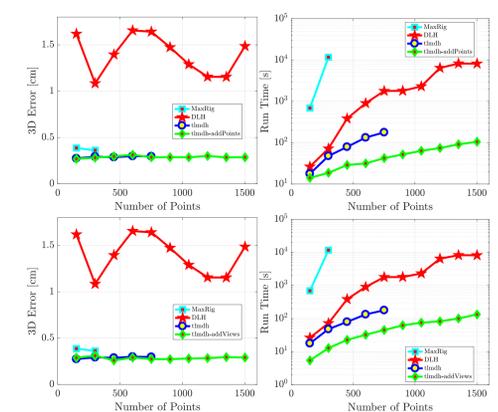
## Incremental Dense NRSfM

Convex formulation to reconstruct additional points consistent with an initial reconstruction.

$$\begin{aligned} \max_{\zeta_i^l, e_{ij}, \alpha} \quad & \alpha \Lambda + \sum_l \sum_{i \in \mathcal{Q}} \zeta_i^l, \\ \text{s.t.} \quad & \|\hat{\mathbf{K}}^{-1}(\zeta_i^l \mathbf{u}_i^l - \alpha \lambda_j^l \mathbf{u}_j^l)\| \leq e_{ij}, j \in \mathcal{N}_p(i), \\ & \|\hat{\mathbf{K}}^{-1}(\zeta_i^l \mathbf{u}_i^l - \zeta_j^l \mathbf{u}_j^l)\| \leq e_{ij}, j \in \mathcal{N}_q(i), \\ & \sum_i \sum_{j \in \mathcal{N}_q(i)} e_{ij} = 1 - \alpha, \end{aligned}$$

Adding views can be treated as an SfT problem.

⇒ Improved runtime and memory efficiency enables dense NRSfM.



## Conclusion

We contribute theoretical insights and practical algorithms for the problem incremental dense NRSfM with unknown focal length. A convex formulation for incrementally adding points and views to an existing reconstruction lays the foundation for dense non-rigid reconstruction. Both contributions have immediate value for practical NRSfM applications.

## NRSfM Toolbox

Check out our NRSfM MATLAB toolbox

- Implementations of our algorithms
- Directly compare to state-of-the-art NRSfM methods



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